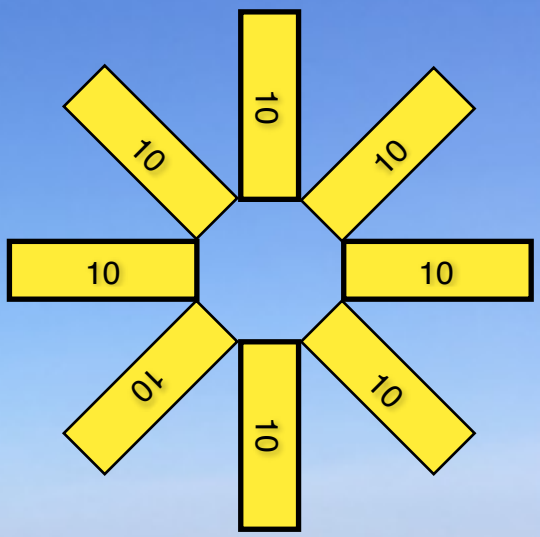
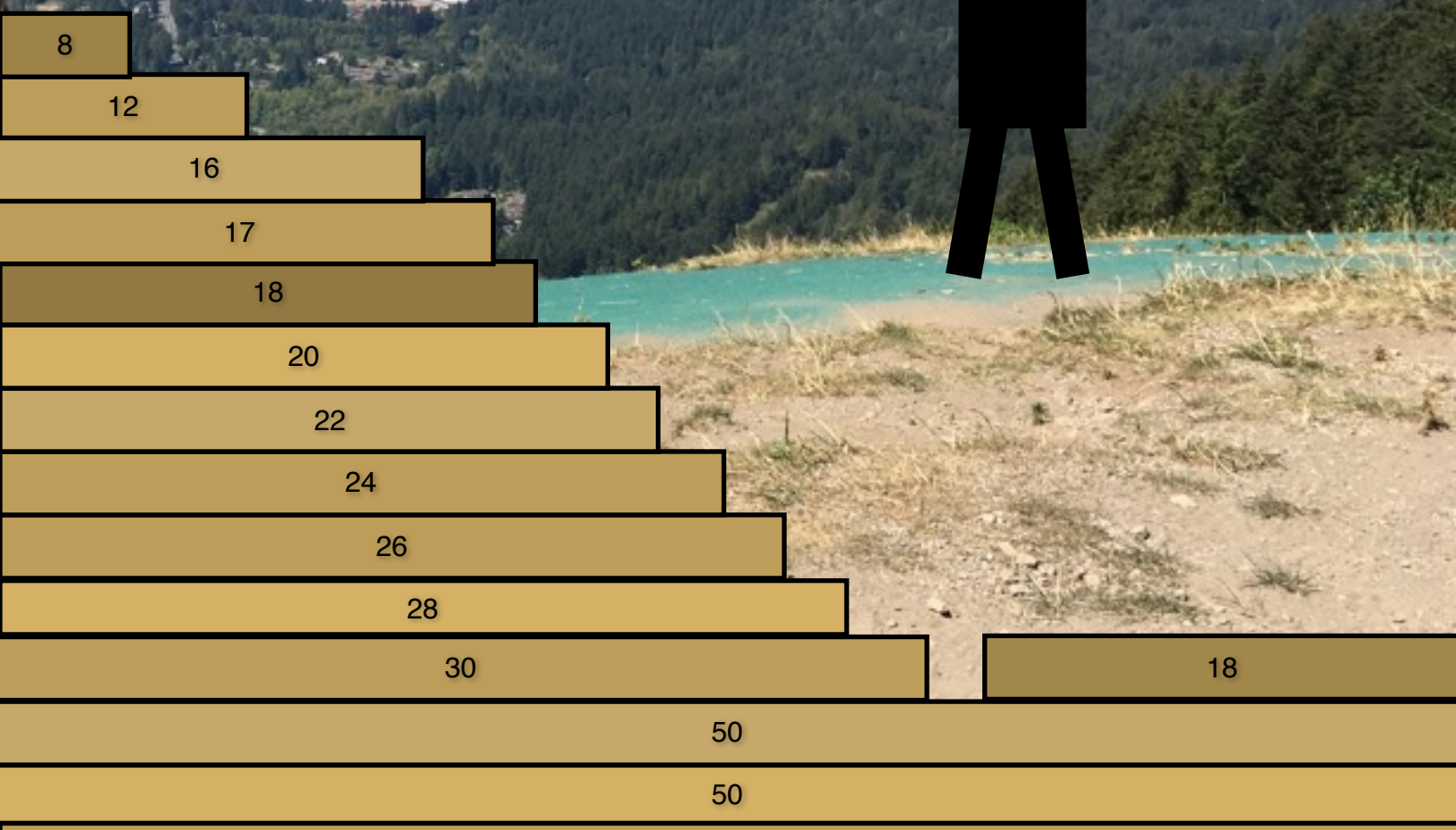
Bar Modeling: Adventures in Algebra





By Alli Domingues, Charlotte Mann, Meg Crenshaw, and Zoe Peterson

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Thank you to our advisor, Deanna Haunsperger
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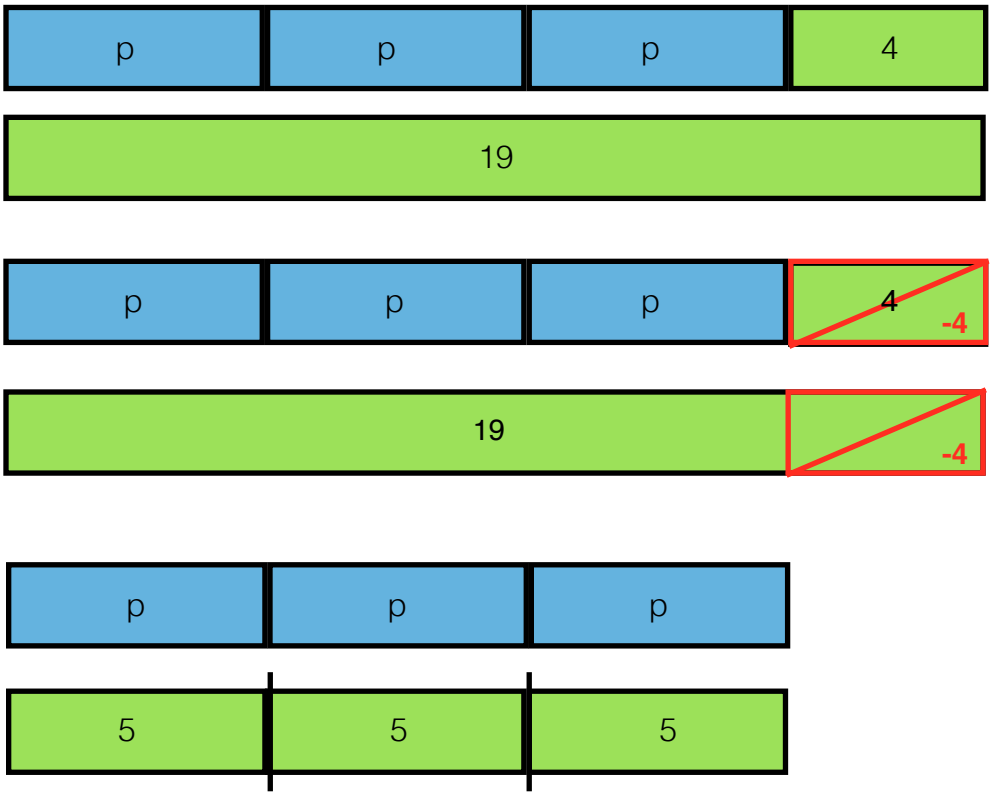
Hi there! We're so excited that you're starting our workbook, "Bar Modeling: Adventures in Algebra".

So what is bar modeling? Bar modeling is a way of drawing pictures to help solve math problems. The best way to explain is through an example. This example might not make sense right now, but soon you'll have the tools to solve problems this way too!

Question: Leon bought three boxes of pizza rolls and a $4 carton of juice. In total he spent $19. How much does one box of pizza rolls cost?



So p= $5!

Bar modeling helps you see the rules of algebra in action!

This workbook will begin with some concepts you might have already seen, but this time you'll learn how to model them with bars.

Now it's time to get started!

Scan here for a video introduction!

How to use this workbook:

- This book starts with **tutorials** to help you learn all the tools you will need to solve problems using bar modeling!

- Before you start on the problems, read all of the tutorials and do the **practice problems**. There are **answers to the practice problems** in the back of the book. It may be more fun to read the tutorials with a partner!

- After you have done all of the tutorials, you are ready to start the **problems**!

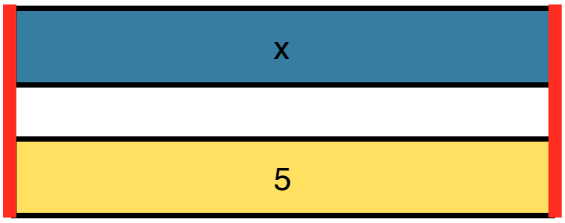
- They may be tricky or confusing at first, but there are examples of solutions throughout the book and **answers in the back of the book.**

Bar Modeling Tutorials

How do we show values are equal?

Imagine that you have been told that x = 5. How could we show this information with blocks?

In the above model the equals sign show that the two blocks are equal. However, it's hard to really *see* they are equal. What if instead we use the *length* of the blocks to show equality?



Now we can see that these two blocks are the exact same length! This indicates that in this example x and 5 are equal.

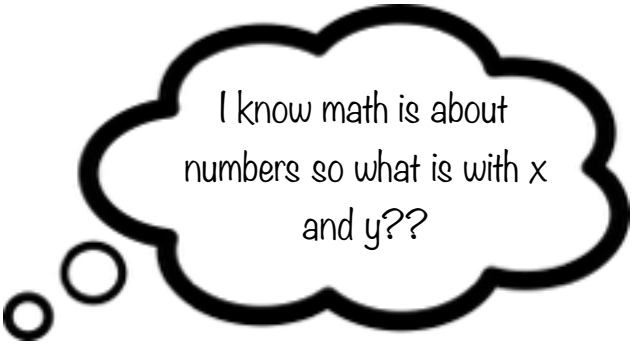
Units

If both x and 5 are counting the same thing, say the number of cows, then this equality makes sense. But what if x is counting cows and 5 is counting shoes

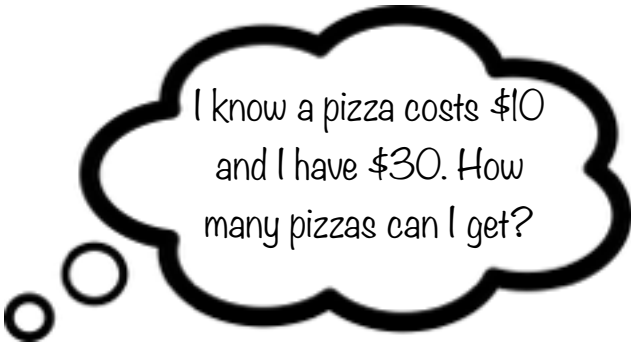
5 cows aren't the same as 5 shoes!

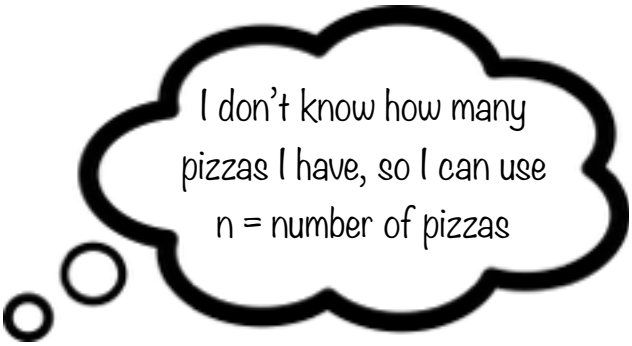
Make sure that when you show equality, you are always comparing the same type of things. Both bars need to have the same **units**.

What's a Variable?



In algebra, we use letters, like x and y, as **variables** to represent numbers that we don't know the value of yet. They are just like numbers, we just don't know what they mean until we solve for them! This is what algebra is all about!

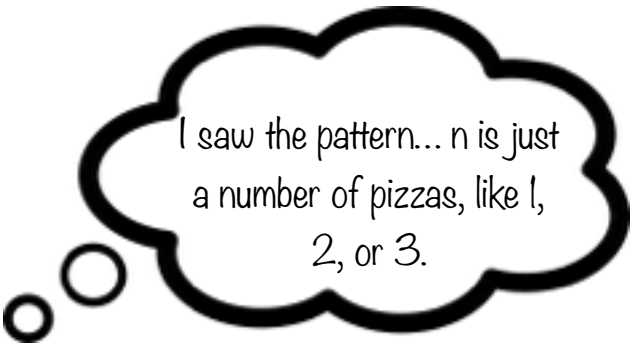




In this case, since we don't know how many pizzas you can get, we can use the variable n = number of pizzas to represent this number we don't know yet.

How much does my pizza order cost?

Number of Pizzas	Cost (in dollars)
1	1x10=10
2	2x10=20
n	nx10 = 10n

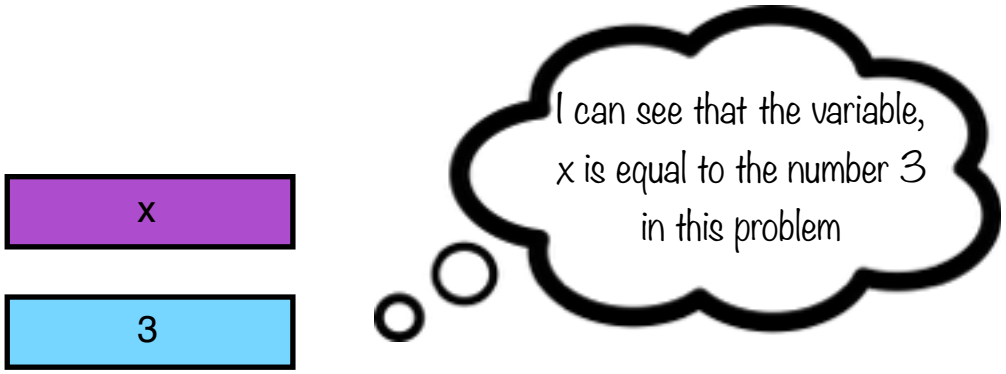


Variables can take on many values. Like I can replace n with 1, 2, or 3 to represent different sized pizza orders.

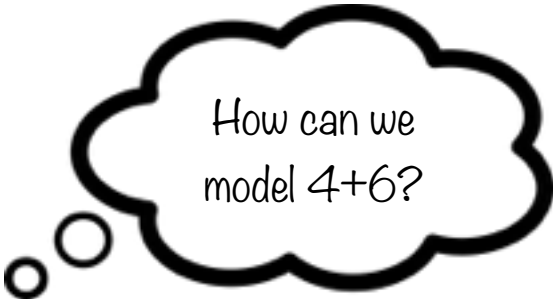
We use variables in bar modeling in the same way, to represent a number. Then we can use bar modeling to figure out the value of the variable!

Since a variable is just standing in for a number we don't know, we use bars to show variables the same way we use bars to show numbers, like this:

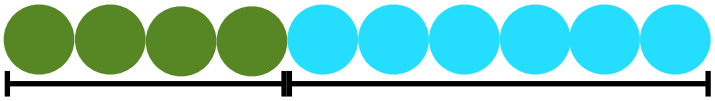
It is a good idea to make your number and variable blocks different colors to keep track of what you know and what you want to find out!



Addition



4 + 6



4 6

When we add 4 circles and 6 circles…

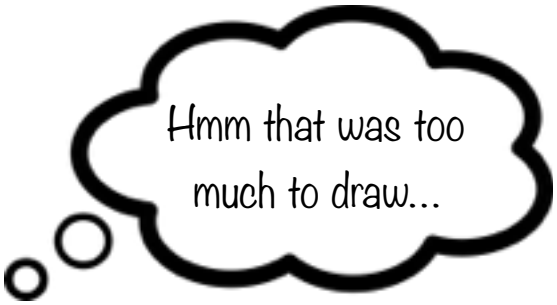
10

… that's 10 circles!

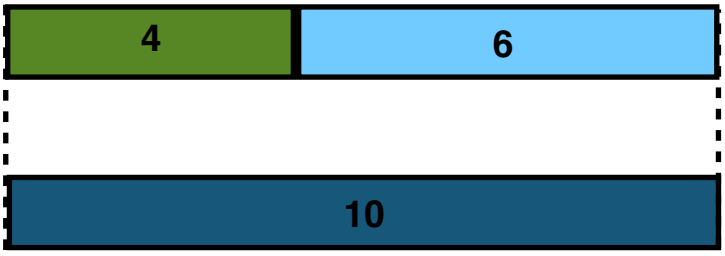
Circles next to each other show addition.

Practice: Model using circles

3 + 5 	1 + 2
7+9	28+5



4 + 6

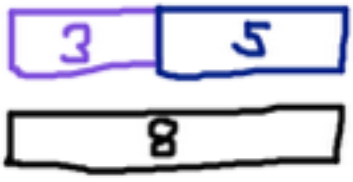


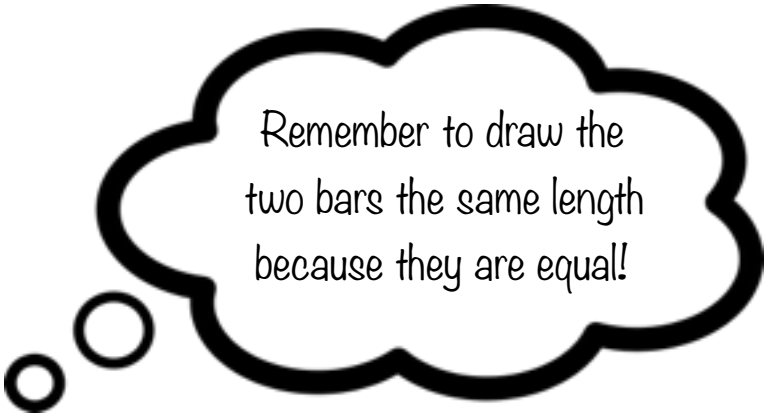
When we draw a 4 block next to a 6 block

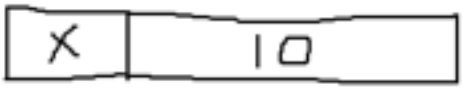
That's the same as one 10 block

Blocks next to each other show addition.

Practice: Model using blocks

3 + 5 	1 + 2
7+9	28+5

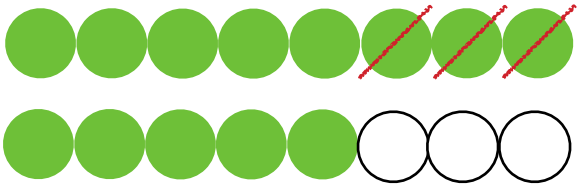


100 + 50	1 + 2 + 3 + 4
10 + 5	10 + 10
x + 10 	y + z + 10
a + a + 5	b + 1 + 2

Subtraction

8 - 3

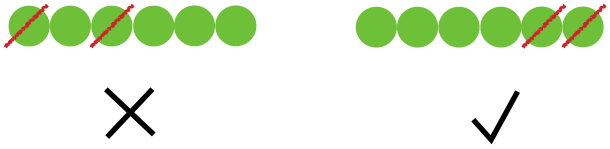
We start with 8 circles



Then we take away 3

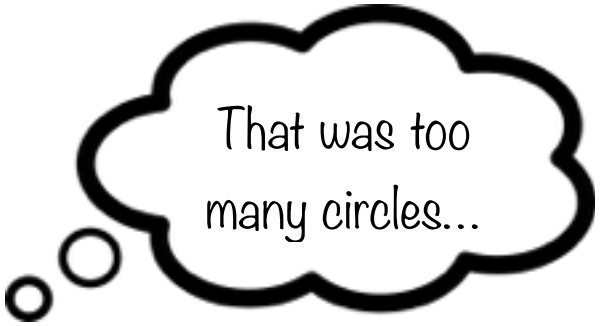
And we are left with 5 circles

Note: Always remove the farthest right circle!



Practice: Model using circles

7 - 4 	9 - 2
13 - 7	25 - 17



8 - 3

We start with an 8 block

Then we draw a subtract 3 block

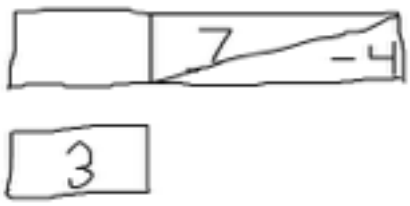
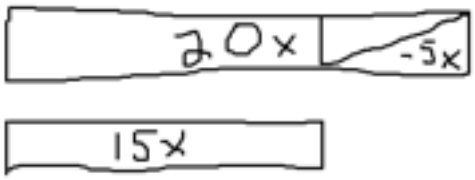
And we are left with a 5 block

Note: Always line up the right side of a subtraction block with the right side of a regular block

Scan here for tutorial video!

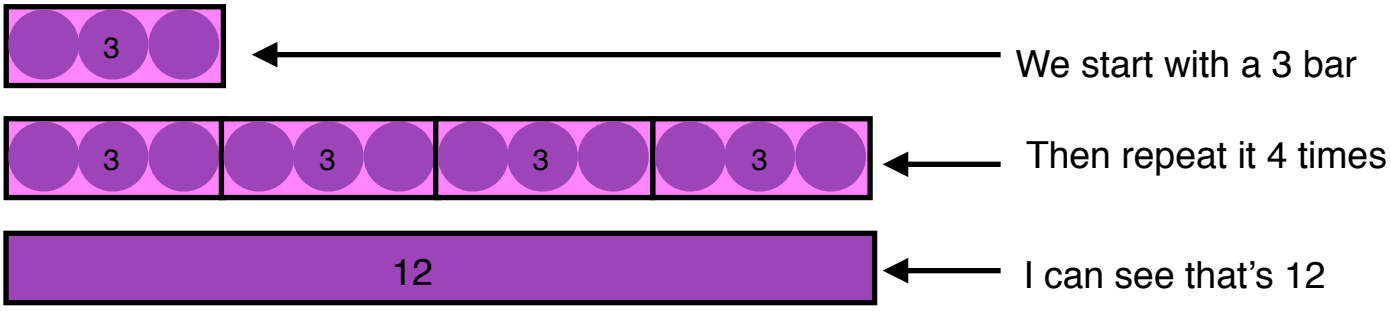
I helped you get started, now you finish the problem!

Practice: Model using bars

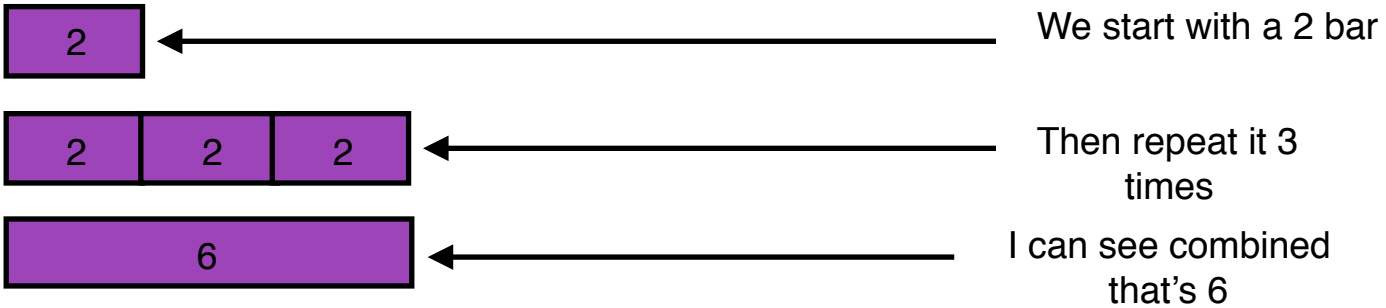
7 - 4 	9 - 2 
13 - 7	25 - 17
18 - 2	20x - 5x 
8y - 4y	30a - 10a

Multiplication

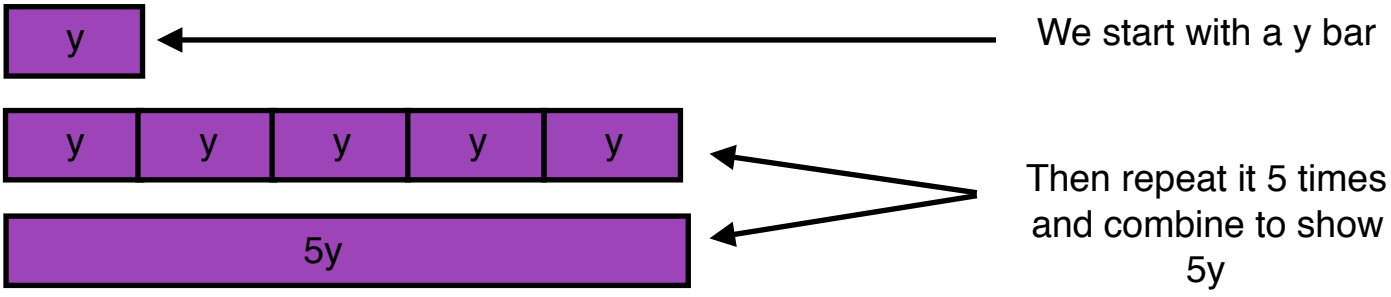
3x4



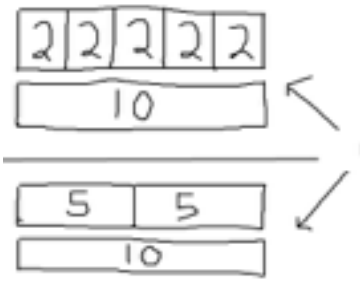
2x3



5y



Practice: Model using bars

2x5	8x6
 Either way works!	
7x3	4x9
29x3	45x6
7y	3z

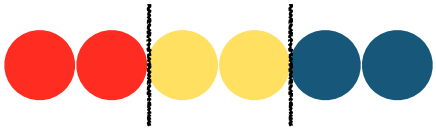
Division

$6 \div 3$

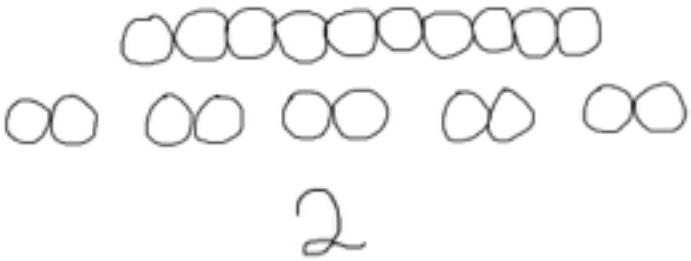
We start with 6 circles

We divide them into 3 piles

There are two circles in each pile, so our answer is 2



Practice: Model using circles

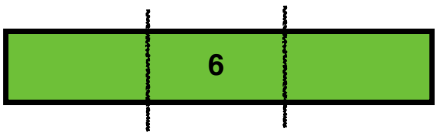
$10 \div 5$ 	$8 \div 4$
$12 \div 4$	$12 \div 3$

6 ÷ 3

We start with a 6 bar

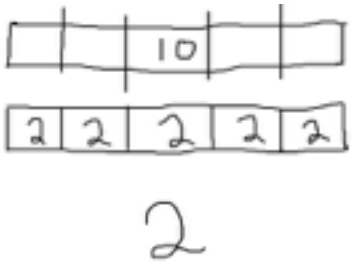
We divide the bar into 3 parts

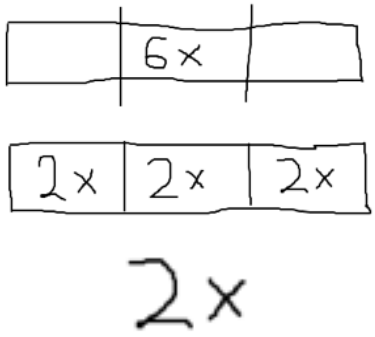
We figure out how big each section is



In this problem, each
bar is of size 2. So our answer is 2!

Practice: Model using bars

10 ÷ 5 	8 ÷ 4
12 ÷ 4	12 ÷ 3

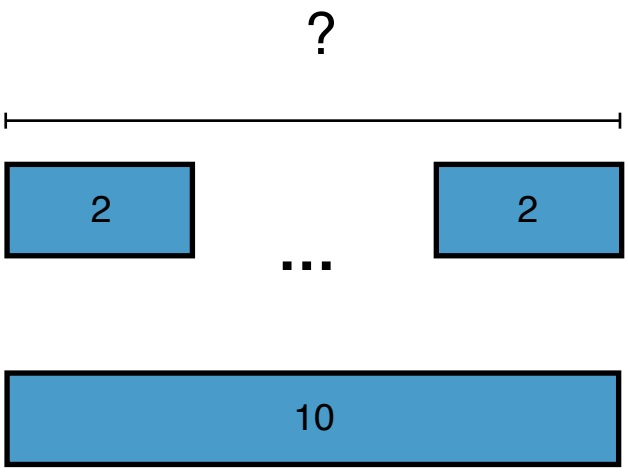
$24 \div 6$	$20 \div 5$
$40 \div 10$	$100 \div 4$
$6x \div 3$ 	$15y \div 5$
$100a \div 10$	$20x \div 2$

Another Way of Viewing Division!

$10 \div 2 = ?$ ← What is 10 divided by 2?

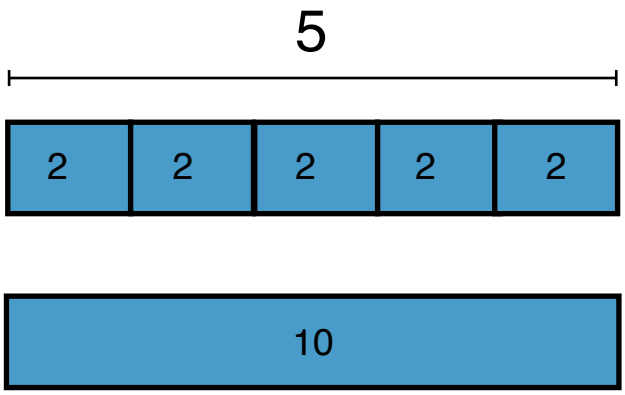
$2 \times ? = 10$ ← How many 2's do we add to have 10?

These two equations are asking the same question!



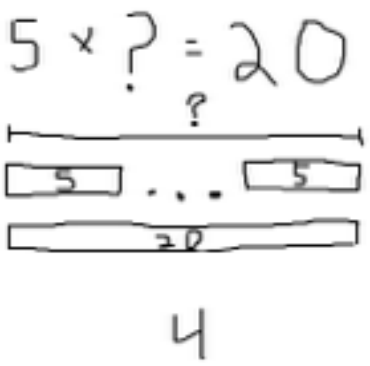
Let's model $2 \times ? = 10$

We use ... to show that we are adding up an unknown number of 2 blocks



Ahhh the ? is 5!
It takes five 2 blocks to make 10.

Practice: Model using bars

20 ÷ 5  4	8 ÷ 4
12 ÷ 4	12 ÷ 3
40 ÷ 10	100 ÷ 4

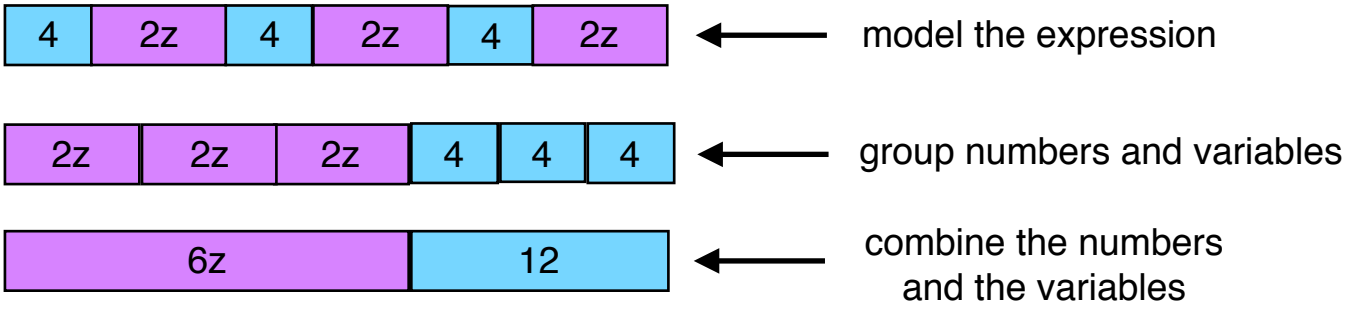
Simplifying with Bars

We now know how to add, subtract, multiply, and divide using bar modeling! And we know what variables are and how to show equality with bar modeling.

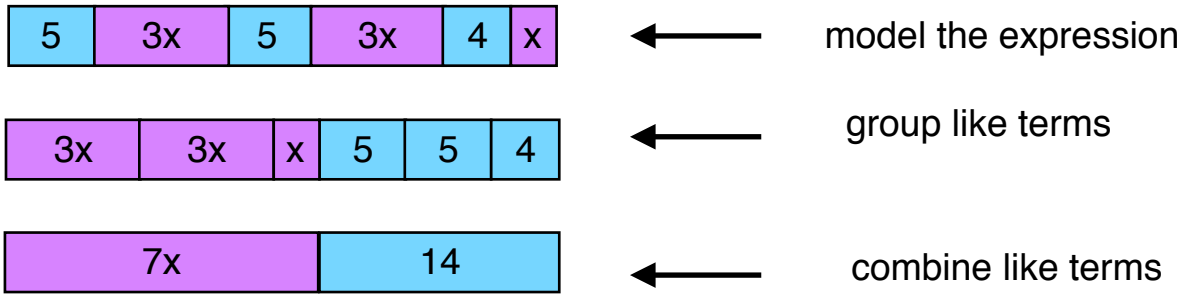
Now it's time to . . .
put it all together!!

Combining Like Terms

Example 1: Simplify 3(4+2z)

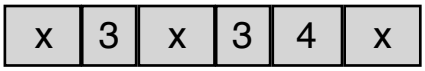
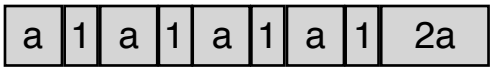
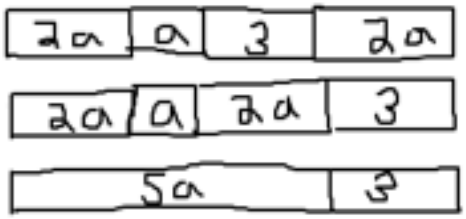


Example 2: Simplify 2(5+3x)+4+x



Practice: Simplify using modeling and combining like terms

4 + 2z + 3z + 1	x + x + 2 + x + 2
4 | 2z | 3z | 1 2z 3z 4 1 5z 5	x | x | 2 | x | 2

2(x + 3) + 4 + x 	4(a + 1) + 2a 
2a + a + 3 + 2a 	2x + 2(x + 3)
2 + 4b + 1 + b	4(2b + 2) + b
3a + b + 2 + a + 2b	x + z + 2y + 2z + y

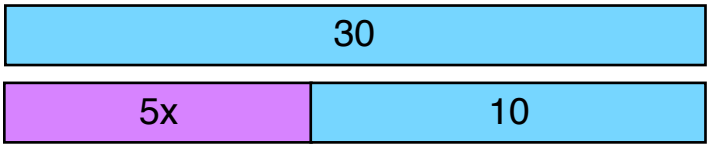
Modeling Equations with Equality

When you're given any equation to solve, you first want to **model** the entire equation.

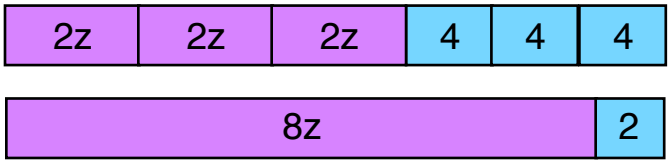
You'll build **two** bars made up of blocks: one bar for each side of the equals sign.

After drawing your model, you want to **check (✓)** to make sure your bars are **equal (=)** in size.

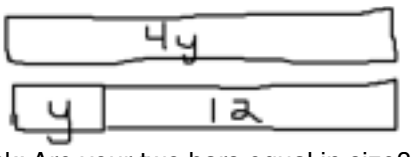
Modeling Example 1: 30=5x+10



Modeling Example 2: 8z+2=3(4+2z)



Practice: Now it's your turn to practice this first step. Model the equation, and then check that your two bars are equal in size.

4y = y +12  Check: Are your two bars equal in size? ✓	20 = 2(q+5) Check: Are your two bars equal in size? ___
a = 2(10)+7 Check: Are your two bars equal in size? ___	4(a+4) = 8(a+1) Check: Are your two bars equal in size? ___

Word Problems

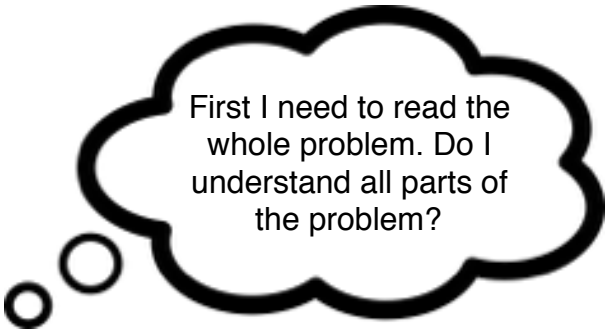
Let's move onto another type of problem: a problem with lots of words and numbers. You may not know where to start, and that's okay. Modeling can help you!

Suppose you are given the following word problem:

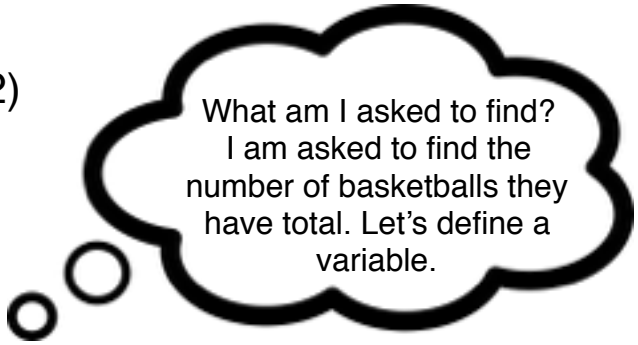
> Zoe and Meg really like basketball. Meg has 8 basketballs, and Zoe has 5 basketballs. How many basketballs do they have total?

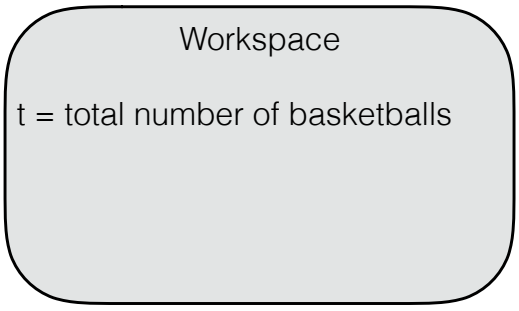
We want to solve this problem by modeling it. Let's see how someone might think about this problem.

1)

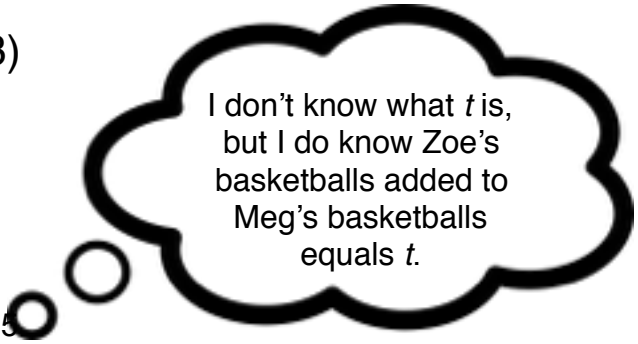


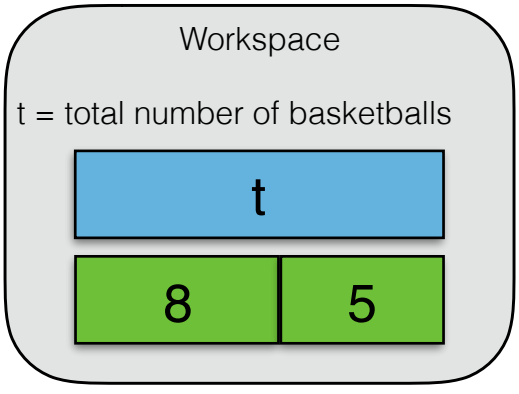
2)





3)





4)

Ah, I can now find t by combining 5 and 8! 13 is our total number of basketballs.

Workspace

t = total number of basketballs

t

13

5)

My model says that *t = 13*. Does that make sense to me?

The total basketballs will be *more* than the number of basketballs Zoe and Meg have individually. 13 is more than 5 and more than 8, so that makes sense!

6)

My answer makes sense! Now I am ready to write my final answer and put a box around it. I can't forget my units!

Workspace

t = total number of basketballs

t

13

13 Basketballs

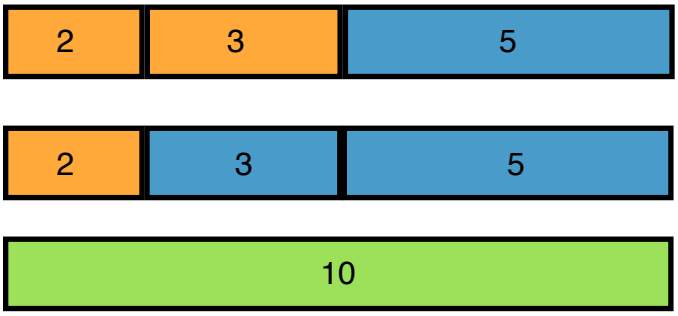
What steps did this student go through to solve this problem?

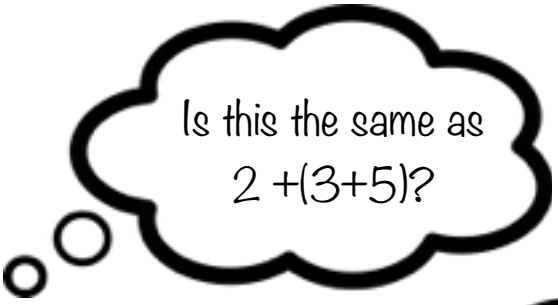
1. **Read** the entire problem and make sure you understand the situation.
2. **Define**: Decide what you're trying to find and define a variable.
3. **Draw** the model for the situation. You will need to decide what parts are equal.
4. **Solve**: Use the model and your tools of addition, subtraction, multiplication, and division to solve the problem.
5. **Check** if your answer makes sense. If it does not, go back and find your mistake.
6. **Write** and box your answer. Don't forget the units!

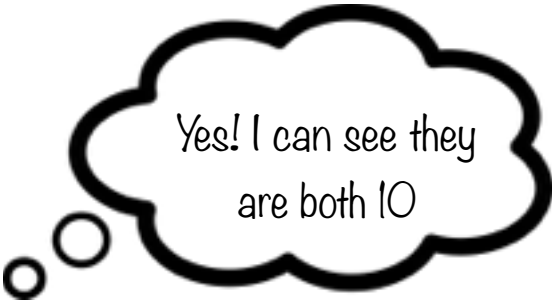
Algebra Concepts

The Associative Property

(2+3)+5







It doesn't matter where the parentheses are, the sum of the numbers is the same. (2+3)+5 = 2+(3+5) = 10.

This is called the **Associative Property** in Algebra. You might see it like this:

(a+b)+c = a+(b+c)

The Associative Property also works for multiplication, so

(a×b)×c = a×(b×c)

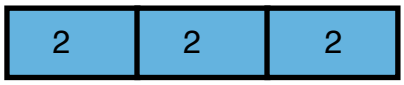
Lets take a look at that together…

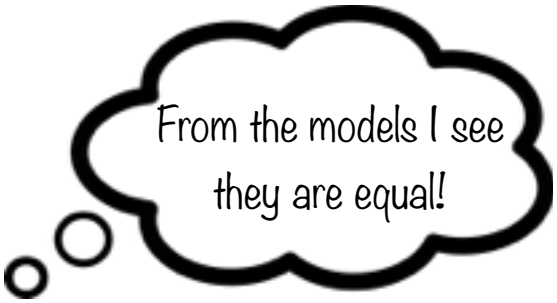
Model 2×(1×3) below:

Now model (2x1)x3:

2x(1x3):

(2x1)x3:

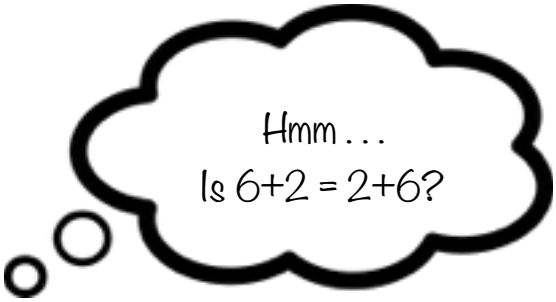




Commutative Property of Addition

Let's model 6+2.

Adding like terms together, we can see that the total is 8.



What if we modeled 2+6 instead?

Surprise! The total is again 8!

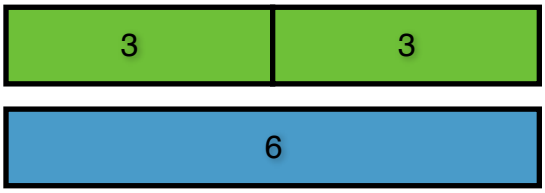
The models above demonstrate the **commutative property of addition**: the order in which numbers are added together does not change the sum! In other words:

a + b = b + a.

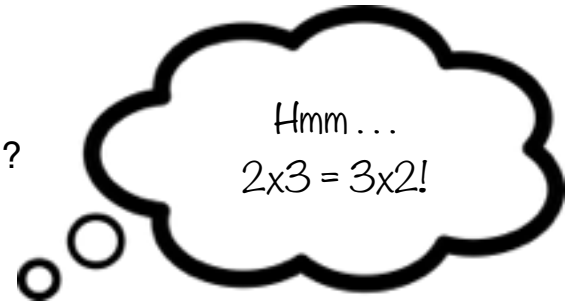
Model 3+5 and 5+3, and show that you get the same result!

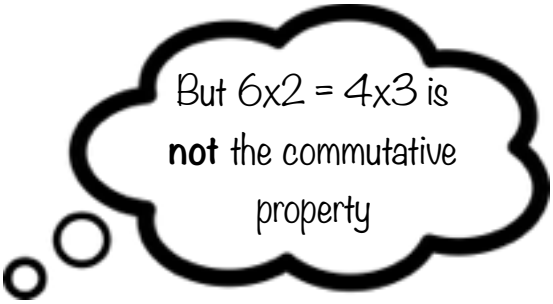
Commutative Property of Multiplication

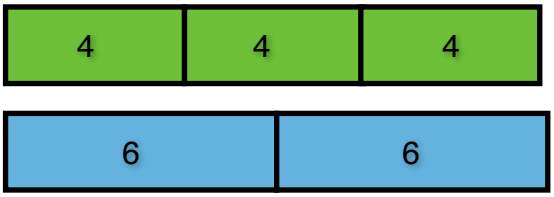
Let's consider the expression 3x2. As we saw in the multiplication section, this can be modeled as two groups of three:



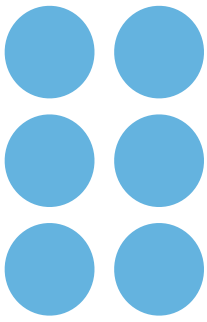
Now what if the expression was written as 2x3?



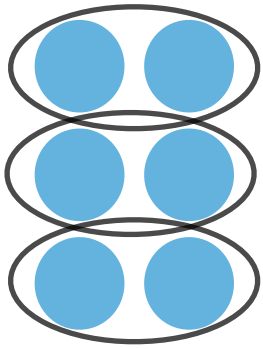




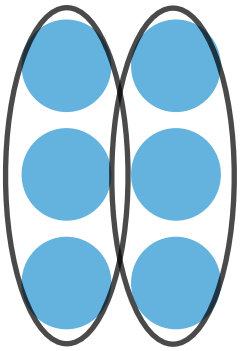
The commutative property applies when you want to **change the order of the numbers** that you are multiplying. Check out the relationship:



3x2 =
2x3



3x2 =
Three groups
of two



2x3 =
Two groups
of three

The models on the previous page demonstrate **the commutative property of multiplication**: the order in which numbers are multiplied does not change the product. In other words:

a x

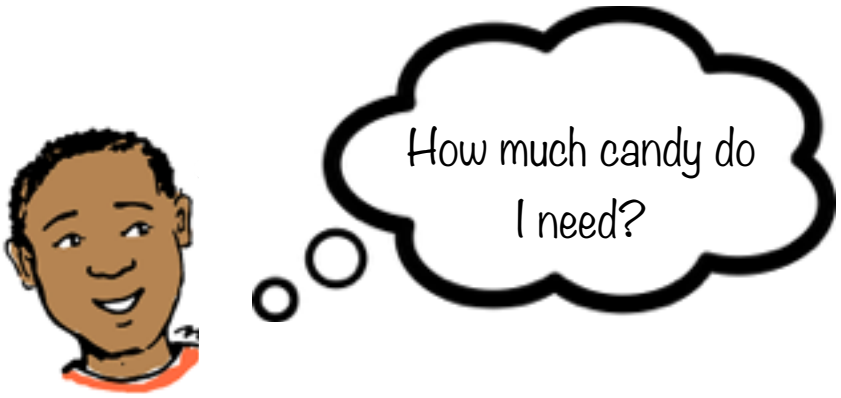
Model 2x4, using either circles or bars.

Now model 4x2, using either circles are bars.

Show that the models of 2x4 and 4x2 are related.
Hint: you can use the strategies on the previous page!

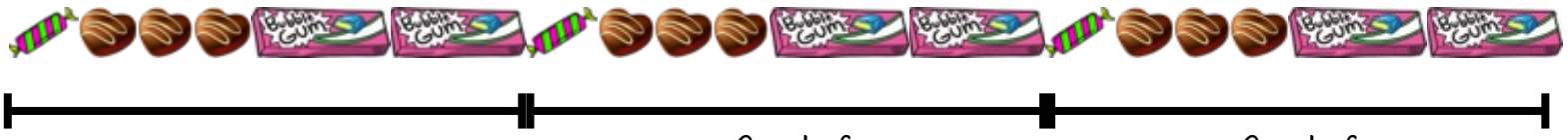
The Distributive Property

I want to give this candy to each of my three friends



$3($  $) = ?$

Let's start by drawing out all of the candy I need



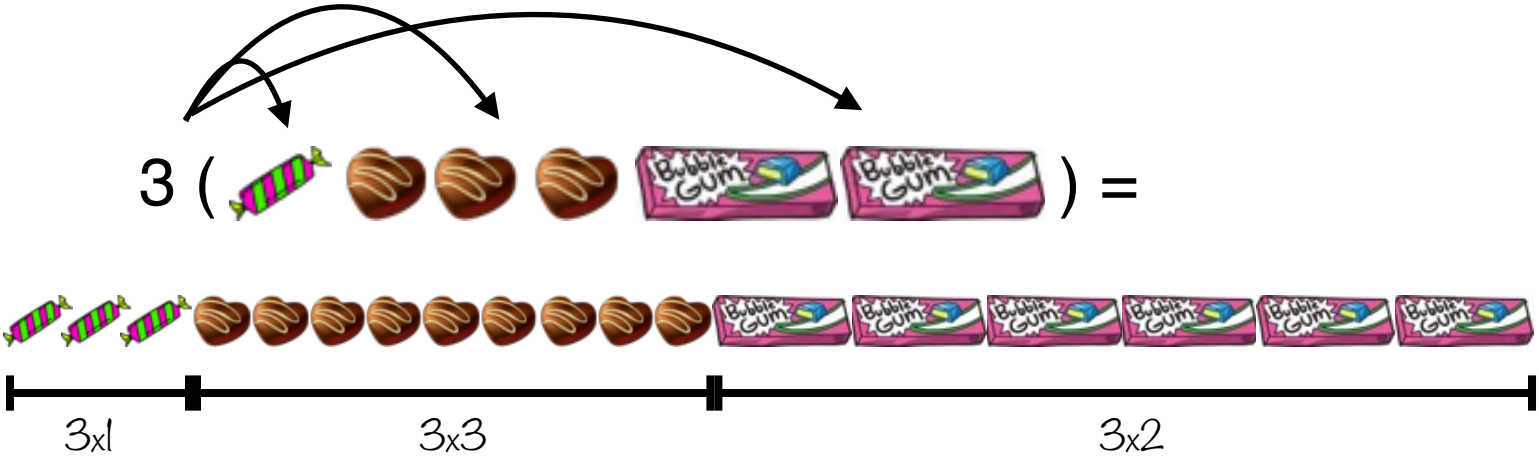
Candy for

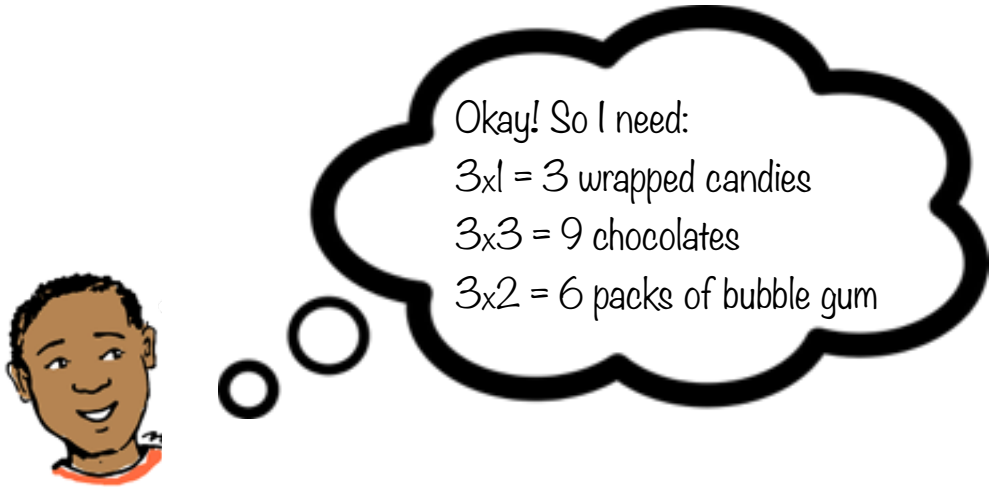
Candy for friend 2

Candy for friend 3

Now let's sort the candy by type

The Distributive Property: Candy



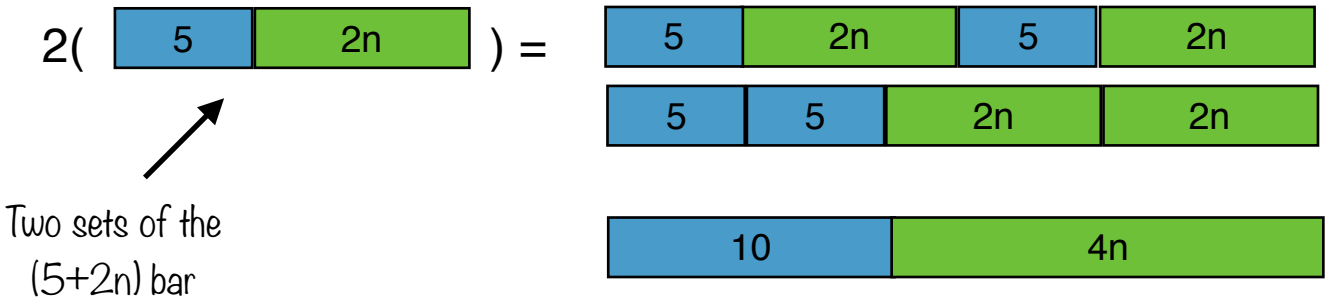


The Distributive Property: Algebra

$$n\,(a + b + c) = n \times a + n \times b + n \times c$$

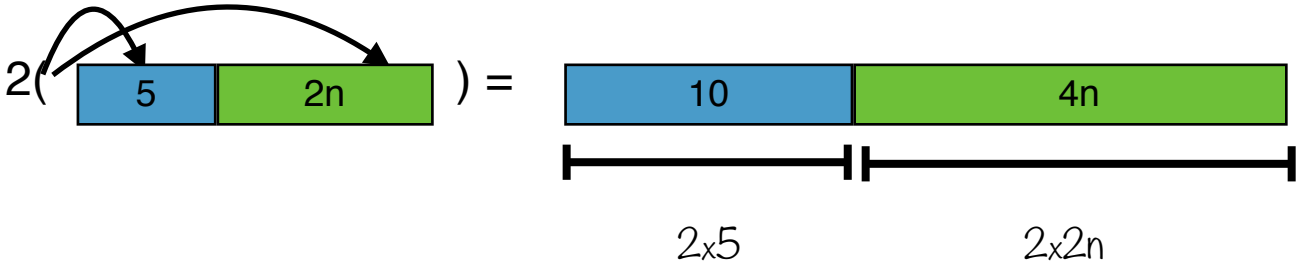
The Long Way

$$2(5+2n) = 10 + 4n$$

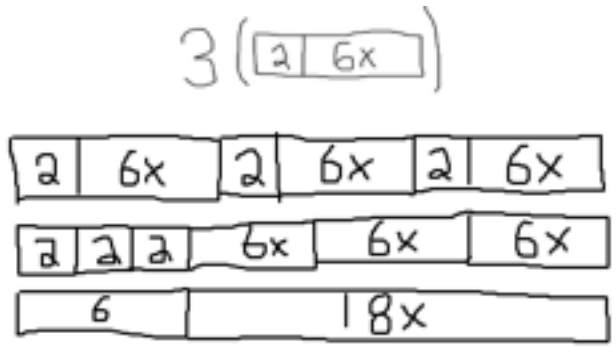


Using the Distributive Property Shortcut

$$2(5+2n) = 10 + 4n$$

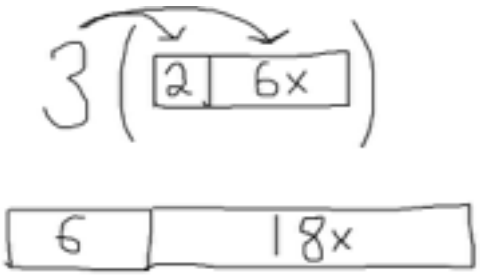
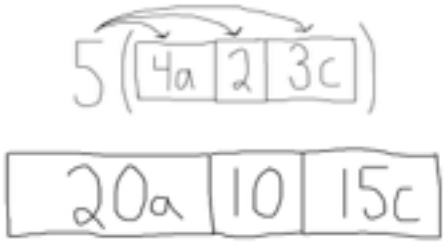


Practice: Model The Long Way

3(2+6x)	2(7+d)
	

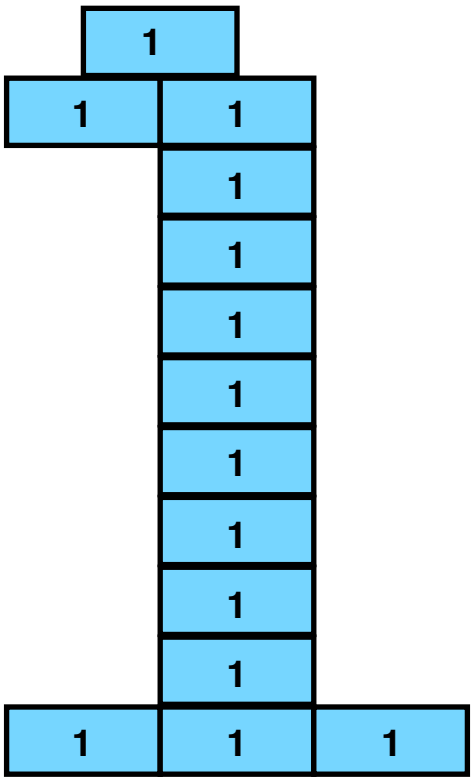
3(2+2d)	2(b+3)

Practice: Model using the Distributive Property

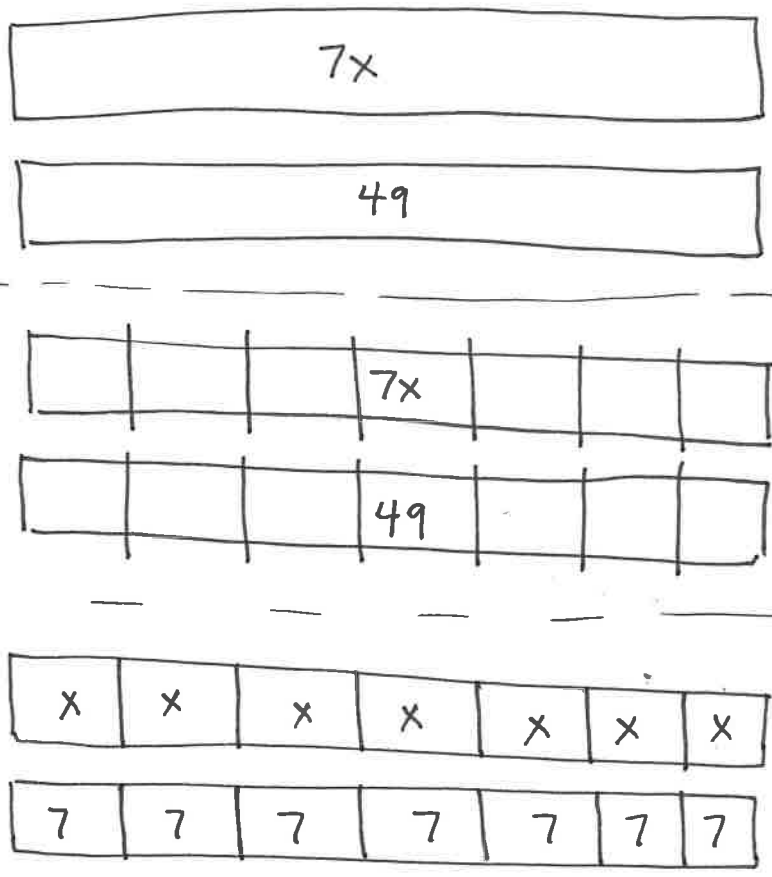
3(2+6x)  	2(7+d)
5(4+3a)	6(4a+2)
5(4a+2+3c)  	3(5x+2y+4)

Problems

Level



Problem: 7x=49



x

7

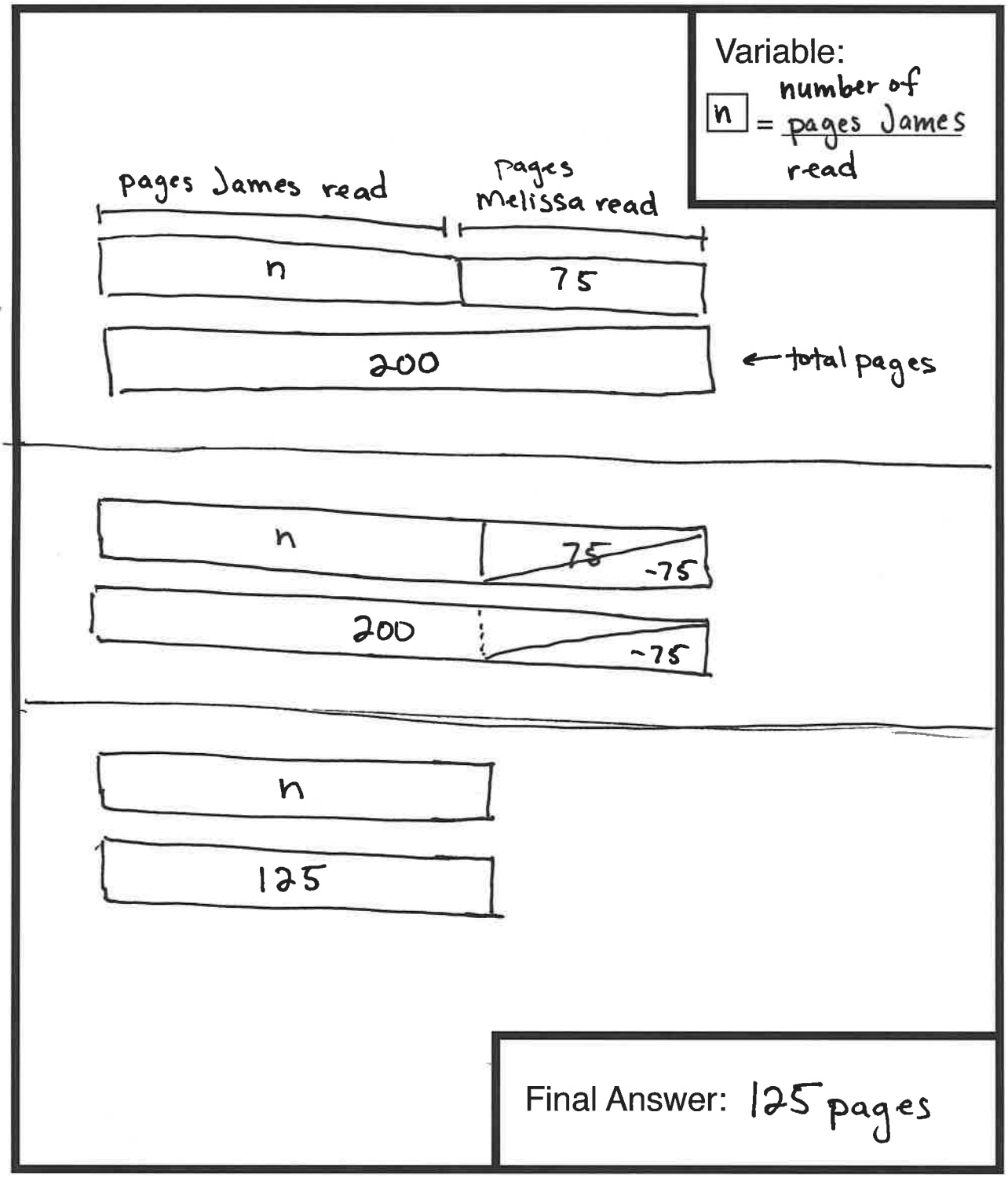
when I divide 7x and 49 into 7 equal parts, I get x and 7. X=7

Final Answer: 7

Problem: x+13=27

Final Answer:

Problem: James and Melissa have a reading assignment to complete. Melissa has read 75 pages. Together they have read a total of 200 pages. How many pages did James read?



Problem: Jake's class is selling boxes of candy for a fundraiser. Their goal is to raise 500 dollars. If they make $2 for each box of candy they sell, how many boxes do they need to sell to reach their goal?

Variable:

☐ = _____

Final Answer:

Problem: Jimmy loves christmas lights! He knows he has 550 total light bulbs. He has 5 strands of lights with the same number of bulbs on each strand. How many lights are on each strand that Jimmy has?

Variable:

☐ = _____

Final Answer:

Problem: Mark bought three notebooks. In total he spent $18. How much did he spend on each notebook?

Variable:

☐ = _____

Final Answer:

Problem: Alicia wants to buy a new scooter for $30. She knows she can earn $5 each time she mows a neighbor's yard. How many yards will she have to mow to afford the scooter?

Variable:

☐ = _____

Final Answer:

Problem: Abdul and Maria are planning a surprise party for their friend. Together they have $12 to spend on balloons. If they buy 4 packages of balloons and spend all their money, how much does a package of balloons cost?

Variable:

☐ = _____

Final Answer:

Problem: Izzy collects marbles. She currently has 12. Her older sister, Adela, tells her that if they combined their collections, they would have 37 marbles total. How many marbles does Adela have?

Variable:

☐ = _____

Final Answer:

Problem: Landon and Chantel are buying supplies for school. Landon spent $3 less than Chantel. Chantel spent $12. How much did Landon spend on school supplies?

Variable:

☐ = _____

Final Answer:

Problem: Christina has to finish a book before her class tomorrow. She has 100 pages left. If Christina knows that she can read twenty pages in a half an hour, how long (in hours) will it take her to finish her reading?

Variable:

☐ = _____

Final Answer:

Problem: Mrs. Dunn's class decides that they want to buy her an end of the year gift. There are 24 students in the class. They want to buy Mrs. Dunn a picture frame that costs $12, how much will each student contribute to split the cost evenly?

Variable:

☐ = _____

Final Answer:

Problem: Felicity has 30 minutes before her bus will come to pick her up. If it takes 9 nine minutes to eat breakfast, how much more time does she have to get ready for school?

Variable:

☐ = _____

Final Answer:

Problem: Ray has to go to soccer practice in an hour. He wants to watch some of his favorite cartoons before leaving. If each episode is twelve minutes, how many episodes can he watch before he needs to leave?

Variable:

☐ = _____

Final Answer:

Problem: Sean's class is going on a field trip with a small participation fee. There are 10 students in his class, who all paid the same amount. All together, they paid $25.00. How much did Sean have to pay?

Variable:

☐ = _____

Final Answer:

Problem: Yolanda is in charge of gathering s'mores supplies for a family camping trip. There are 5 people in Yolanda's family and the package has 15 segments of chocolate. How many segments of chocolate does each person get to make s'mores?

Variable:

☐ = _____

Final Answer:

Problem: Hernando can't remember how much money he had in his wallet before lunch. He knows he spent $8 on lunch at Chipotle, and he has $13 left in his wallet now. How much money did he have before lunch?

Variable:

☐ = _____

Final Answer:

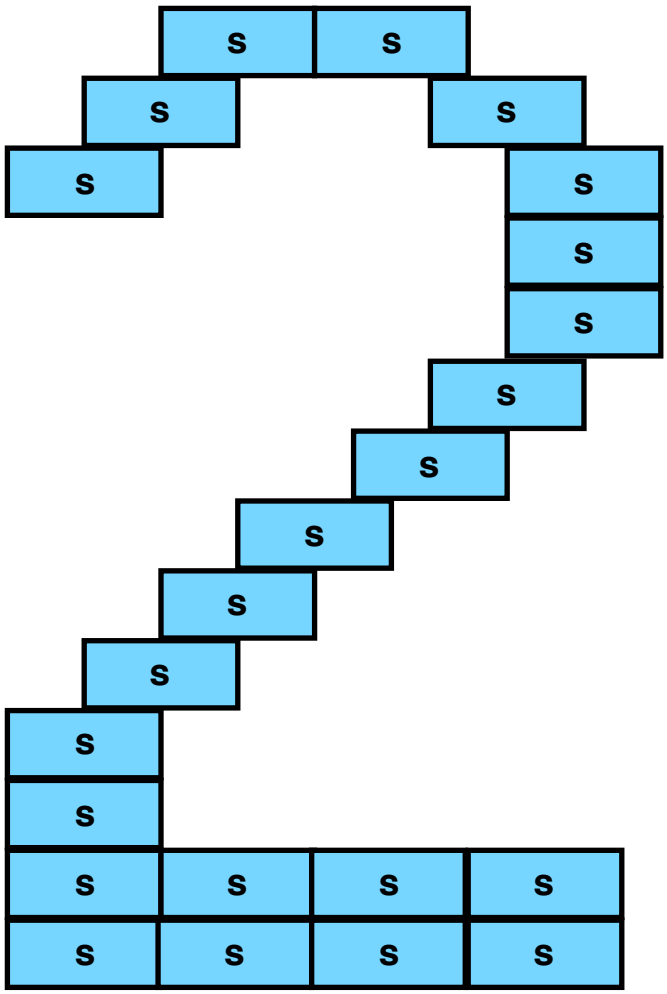
Problem: Jason had 131 dollars to spend on 6 books. After buying the books he had 11 dollars. Each book costs the same amount. How much did each book cost?

Variable:

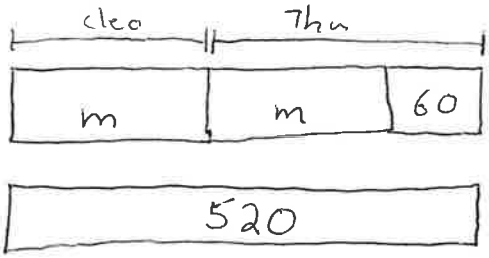
☐ = _____

Final Answer:

Level

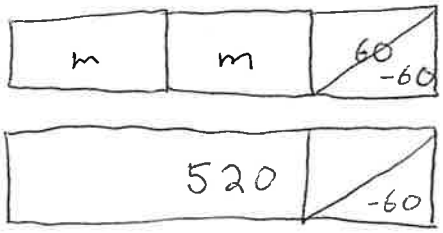


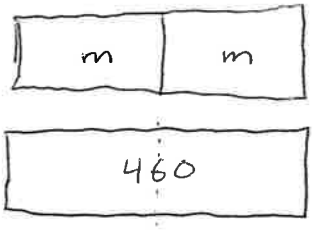
Problem: Thu and Cleo are sharing the driving on a 520 mile trip. If Thu drives 60 miles more than Cleo, how far did each of them drive?



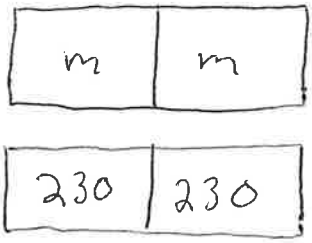
Variable:

m = miles driven by Cleo





230 + 60 = miles driven by Thu



Final Answer: Cleo drove 230 miles
Thu drove 290 miles

Problem: Linnea is planning on cooking dinner with three of her friends. Linnea's mom agrees to contribute $12 towards the cost of ingredients, and Linnea and each of her friends agree to split the remaining cost equally. If the ingredients cost $24, how much will Linnea end up spending?

Variable:

☐ = _____

Final Answer:

Problem: 75 = 8x + 11

Final Answer:

Problem: You ride a taxi for 17 minutes, and you know that they charge a base fee (an amount charged before any minutes have gone by) and $2 per minute. After the trip, $40 dollars total is charged. How much was the base fee?

Variable:

☐ = _____

Final Answer:

Problem: The sum of two numbers is 12. One number is 4 more than the other number. What is the value of the larger number?

Variable:

☐ = _____

Final Answer:

Problem: 23 = 5x - 7

Final Answer:

Problem: The school is putting in a new row of lockers! You have 100 feet of space, and each locker is 1 ½ feet wide. There needs to be 5 feet of extra space at each end of the row. How many lockers can you put in?

Variable:

☐ = _____

Final Answer:

Problem: You are at a school assembly and getting bored. You know the next class is supposed to start in 40 minutes, and the teachers usually allow 10 minutes after assembly for you to get back to class. You estimate each speaker is about 6 minutes. If there are only speakers remaining in assembly, how many more speakers are there?

Variable:

☐ = _____

Final Answer:

Problem: $4x + 7 = 19$

Final Answer:

Problem: I am some number. Multiply me by 6, and then add 4 and you get 10. What number am I?

Variable:

☐ = _____

Final Answer:

Problem: Last week Javier had twice as many stickers as Daniel. Then Daniel received 12 stickers for his birthday. Together they now have 90 stickers. How many stickers did Daniel have last week?

Variable:

☐ = _____

Final Answer:

Problem: 131 = 6x + 11

Final Answer:

Problem: Sunny Hill Farms and Babbling Brooks Farms both raise hens. Sunny Hill Farms has 35 hens. Babbling Brooks Farms has 15 hens. If together the farms have 500 eggs at the end of the week, and each hen laid the same number of eggs, how many eggs did each hen lay last week?

Variable:

☐ = _____

Final Answer:

Problem: Felix wants to buy a new pair of shoes that cost $30. He has $15 now, and knows he can earn $5 a week helping his grandma with yard work. In how many weeks can Felix afford the shoes?

Variable:

☐ = _____

Final Answer:

Problem: $520 = x + (x + 60)$

Final Answer:

Problem: Jason had 131 dollars to spend. After buying 5 books he had 11 dollars left. Each book cost the same amount. How much did each book cost?

Variable:

☐ = _____

Final Answer:

Problem: $36 = 12 + 4x$

Final Answer:

Problem: 50 = 10 + 4x

Final Answer:

Problem: 12x3 + x = 43

Final Answer:

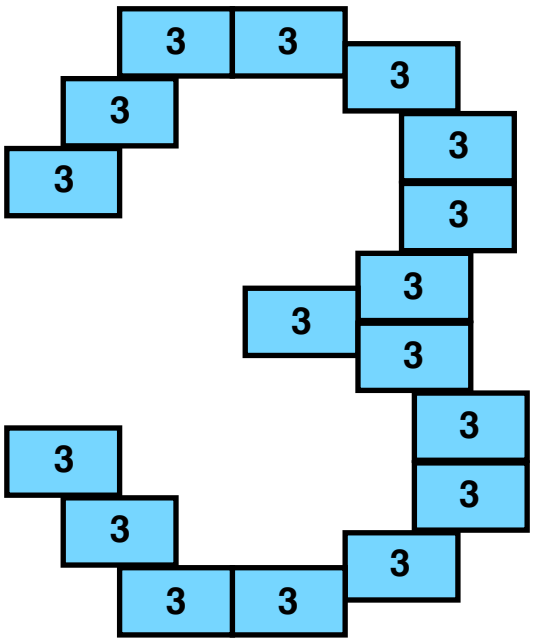
Problem: $12 = x + (x - 4)$

Final Answer:

Problem: $6x + 4 = 10$

Final Answer:

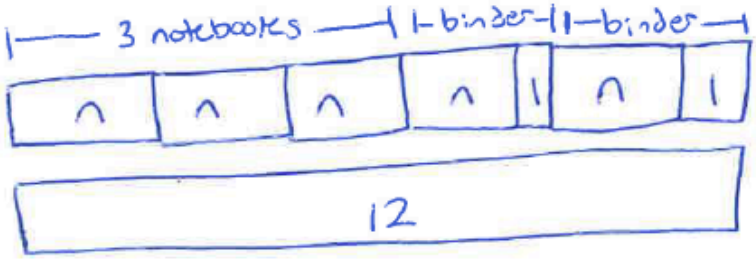
Level

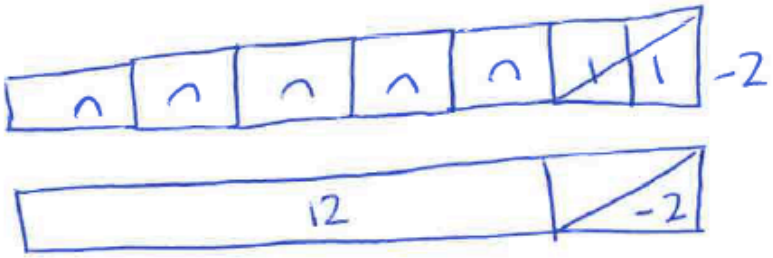


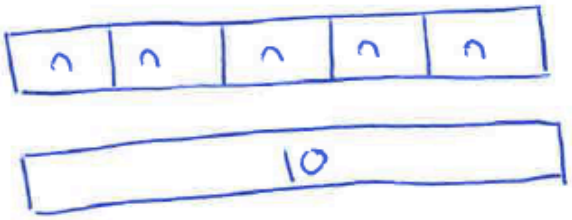
Problem: Alicia bought 3 notebooks and 2 binders, and her sister agreed to pay for the notebooks. Unfortunately, Alicia can't remember the cost of the notebooks! She does remember that each binder cost $1 more than each notebook and that she spent $10 total. How much does Alicia's sister need to pay Alicia?

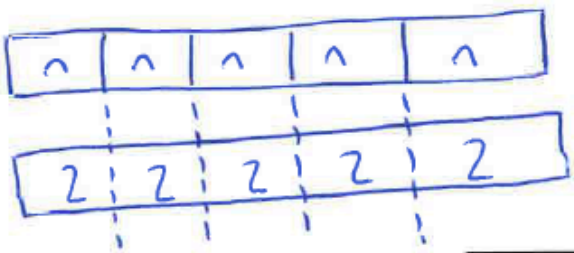
Variable:

$\square$ = cost of one notebook









So each notebook is $2. Then her sister pays for 3, meaning she pays $6!

Final Answer: 6 dollars

Problem: Philippe and Finn go to see a movie. Each buys a ticket for $7 and a slushie, spending $18 together. How much does one slushie cost?

Variable:

☐ = _____

Final Answer:

Problem: $3(x + 2) + 4x = 27$

Final Answer:

Problem: $4x + 2x = 24$

Final Answer:

Problem: Jordan goes to the grocery store and buys one bag of chocolate, one bag of caramel, and one bag of lollipops. A bag of chocolate costs two dollars more than three times as much as a bag of caramel, and a bag of lollipops costs as much as buying a bag of caramel and a bag of chocolate. If Jordan spends 20 dollars for the three bags, how much does a bag of chocolate cost?

Variable:

☐ = _____

Final Answer:

Problem: Kim is three years older than her sister Jess and half as old as her cousin Lexi. If they add their ages together, they get 37. How old is Jess?

Variable:

☐ = _____

Final Answer:

Problem: Jeremiah had a busy Saturday morning! Starting at 9:00am, he cleaned his bedroom for a while. Then he helped his mom clean the garage for four times as long as he had spent cleaning his room. Finally he cooked breakfast with his sister for ten minutes fewer than the time he had spent cleaning his room, until 9:56 am. How many minutes did Jeremiah spend cleaning his room this morning?

Variable:

☐ = _____

Final Answer:

Problem: Izumi is running the mile (4 laps) at a track meet. She knows that she can run her first lap in 75 seconds. Izumi also knows that her second and third laps are the same speed, while her final lap is normally 9 seconds faster than her third lap. If she wants to finish in 6 minutes, how fast should her second lap be? (Note: there are 60 seconds in 1 minute)

Variable:

☐ = _____

Final Answer:

Problem: $5(x+5) - 2(2x+4) = 18$

Final Answer:

Problem: Jamal has three reading assignments to complete. In total he has to read 70 pages. Assignment 2 is twice as long as assignment 1, and assignment 3 is four times long as assignment 1. How many pages is his shortest assignment?

Variable:

☐ = _____

Final Answer:

Problem: Malia and Megan ordered 3 pizzas and each pizza had 8 slices. Their friend Nilver ate 4 slices of pizza, their friend Shayna ate twice as many pieces as Nilver. Malia and Megan ate all of the remaining slices. How many slices did Malia and Megan eat?

Variable:

☐ = _____

Final Answer:

Problem: $2(2x + 5 + x) + 10 - x = 60$

Final Answer:

Problem: Shailee and Sofia are on a road trip to see their grandparents. They drive for a while before stopping for lunch. After that, they drive again for 3 hours before getting gas. Before reaching their grandparents house, they drive 1 hour less than twice as long as they drove before the first stop. In total, they drove 14 hours on their trip. How long did they drive before the first stop, for lunch?

Variable:

☐ = _____

Final Answer:

Problem: Marcos picked up three books from the library. *The Uglies* is twice as long as *A Wrinkle In Time*, and *A Wrinkle In Time* has forty pages more pages than *The BFG*. Altogether, the three books have 960 pages. How many pages long is *The BFG*?

Variable:

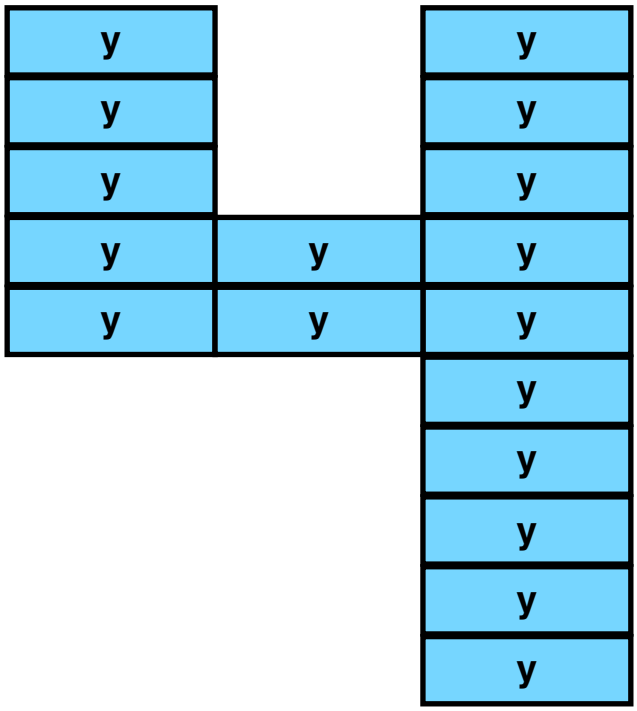
☐ = _____

Final Answer:

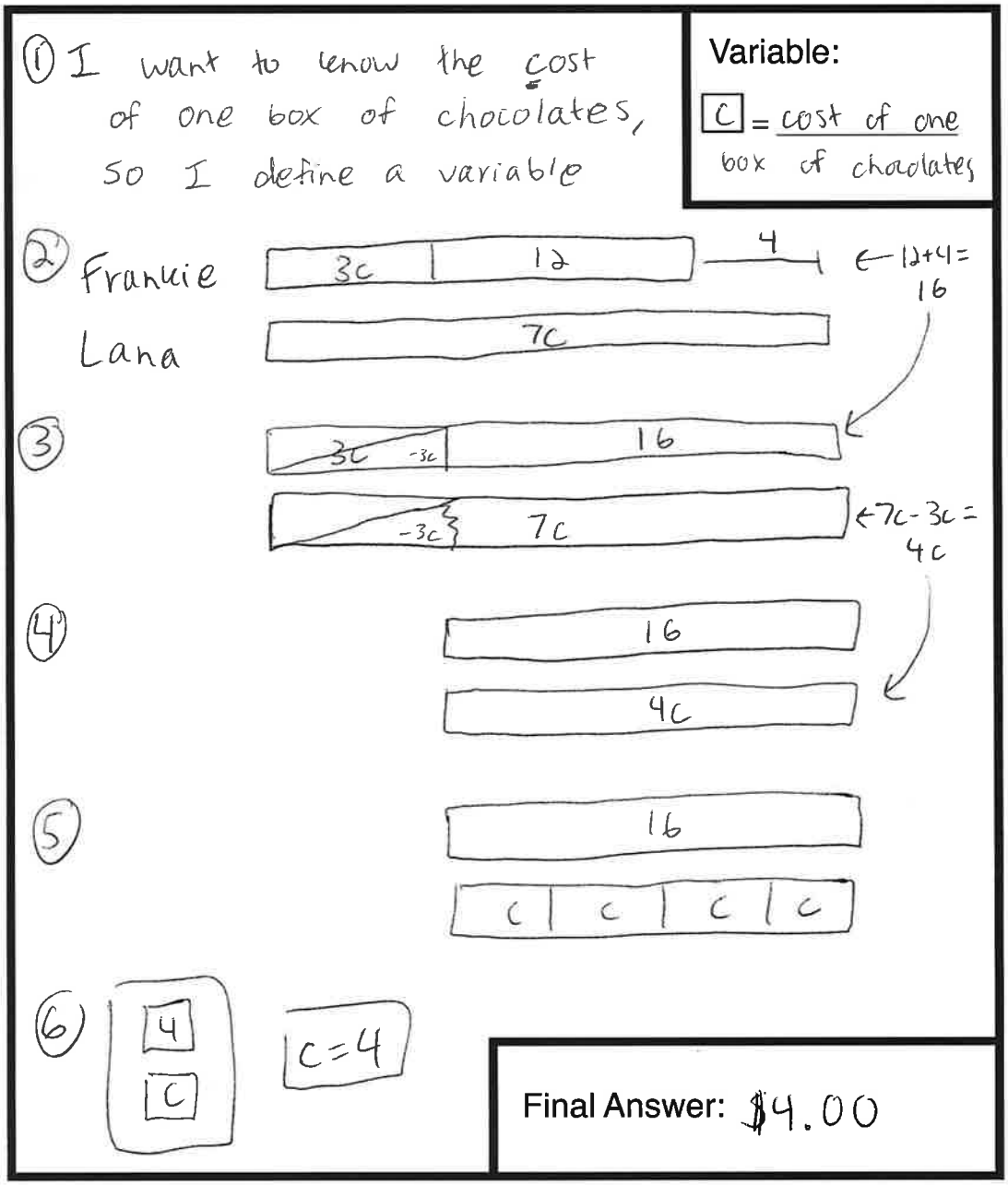
Problem: $3(x + 8) + 2(x + 1) = 36$

Final Answer:

Level



Problem: Frankie and Lana are both selling candy for a school fundraiser. Frankie sells three boxes of chocolate in addition to $12 worth of hard candies. Lana sells seven boxes of chocolate and brags that she has earned $4 more than Frankie. How much does each box of chocolates cost?



Problem: Monica and Bridget bought the same binders for school, and after filling them with school supplies, they weigh the same amount! Monica's binder contains a pencil pouch that weighs 2 pounds and 3 notebooks. Bridget's binder contains a stapler that weighs 1 pound, a pack of crayons that weighs 2 pounds, and 2 notebooks. How much does 1 notebook weigh?

Variable:

☐ = _____

Final Answer:

Problem: $3x+1=x+9$

Final Answer:

Problem: Jimmy always goes on runs that are the same distance. Last week he went on a run where he ran his favorite trail 2 times and then ran 3 miles to the park. Today, he ran his favorite trail 3 times and then ran 1 more mile. How many miles is his favorite trail?

Variable:

☐ = _____

Final Answer:

Problem: Samantha and Carlos wore braces for the same number of years. Samantha can't remember how many years her doctor said she would need braces for, but she knows she had braces for 3 years longer than the doctor expected. Carlos had braces for twice as long as Samantha was supposed to. How long was Samantha supposed to have braces for?

Variable:

☐ = _____

Final Answer:

Problem: $2t + 15 = 4t + 5$

Final Answer:

Problem: In middle school, Frank and AJ went to the same number of dances. Frank went to 2 dances in 6th grade, and 3 dances in 7th grade. AJ didn't go to any dances 6th grade, and went to to 1 dance in 7th grade. In 8th grade AJ went to three times as many dances as Frank. How many dances did Frank go to in 8th grade?

Variable:

☐ = _____

Final Answer:

Problem: Samantha and Elsa have the same number of photos on their phones, and all of their pictures are either selfies or pictures of their pet. Samantha's phone has 5 times as many selfies as Elsa's. Elsa's phone has 20 pictures of her dog, and Samantha's phone has 4 pictures of her cat. How many selfies does Elsa have on her phone?

Variable:

☐ = _____

Final Answer:

Problem: $3(p + 3) + 1 = p + 24$

Final Answer:

Problem: Every Friday, Lola has her friends over and they eat pizza rolls. Last week her friends made 3 boxes of pizza rolls and ate 5 pizza rolls that were leftover in the fridge. This week they ate 4 boxes of pizza rolls and ate 3 more rolls than they ate last week. How many pizza rolls are there in one box?

Variable:

☐ = _____

Final Answer:

Problem: Frank has eleven siblings, and they all like to eat cheese sticks. Their dad buys the same number of cheese sticks every week. Last week, Frank ate many cheese sticks, and each of his siblings ate three cheese sticks. This week, Frank and his sister Sofia *each* ate one more than Frank had eaten the week before, and each of their *other* siblings ate two cheese sticks. How many cheese sticks did Frank eat last week?

Variable:

☐ = _____

Final Answer:

Problem: $y + 3 = 10y - 15$

Final Answer:

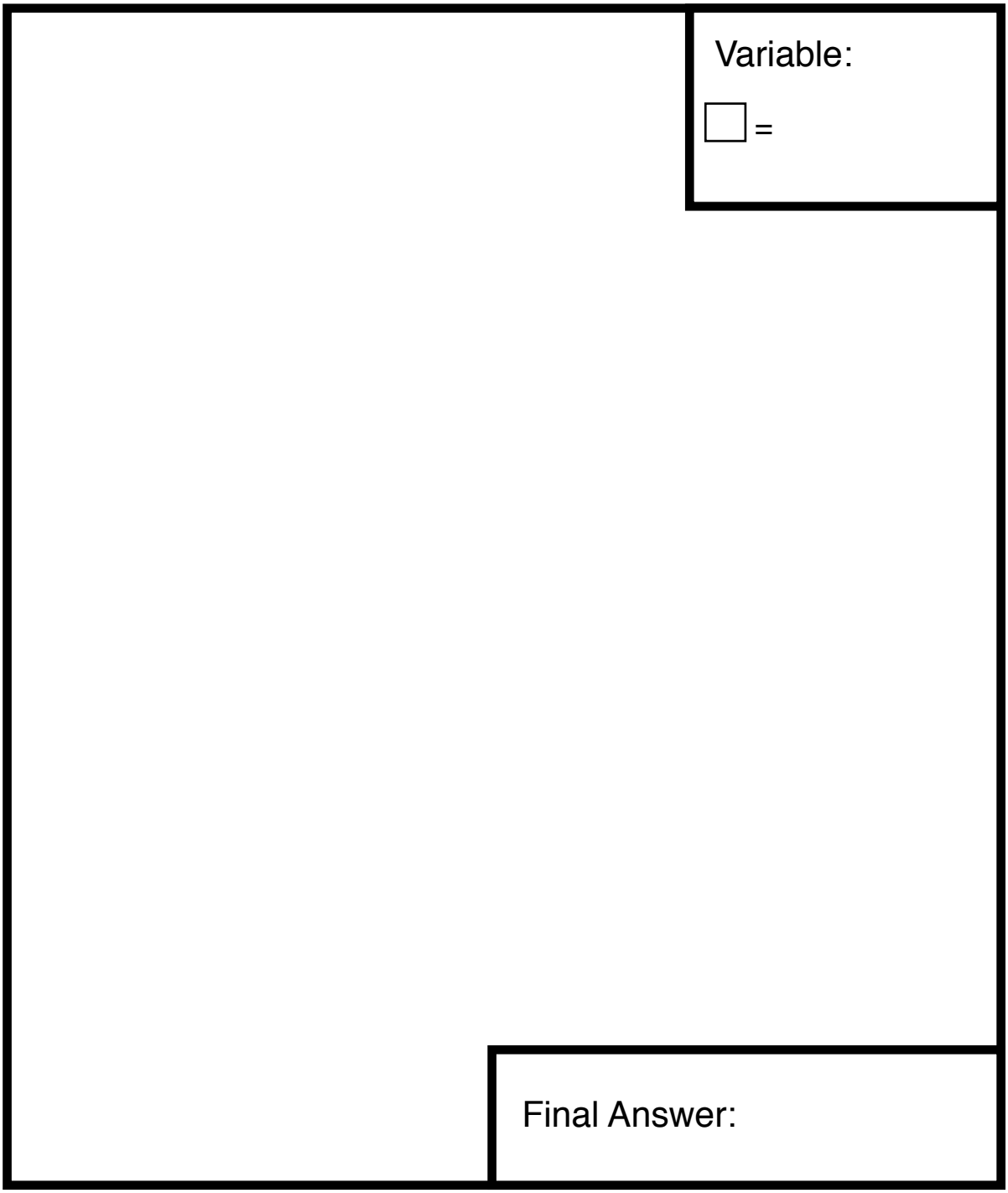
Problem: Gretchen plays the clarinet, and her teacher has a required amount of time that a practice session is supposed to last. Last week she practiced the required amount of time 6 times, and practiced for 30 extra minutes on Thursday. This week she practiced the required amount 5 times and practiced 90 minutes less this week than last week. How long does Gretchen's teacher require that a practice session last?

Variable:

□ =

Final Answer:

Problem: Ron and Harry love to tell jokes. On Tuesday Ron told 3 jokes in each class period and Harry told 5 jokes in each class period. Ron also told 8 jokes during lunch, and Harry told 2 jokes during lunch. If they only told jokes during class and at lunch, and they both told the same number of jokes on Tuesday, how many class periods were there on Tuesday?

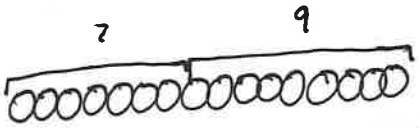


Problem: $7(x+1) = 4x + 43$

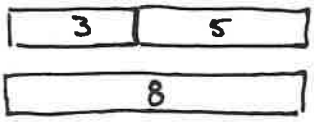
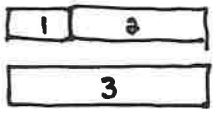
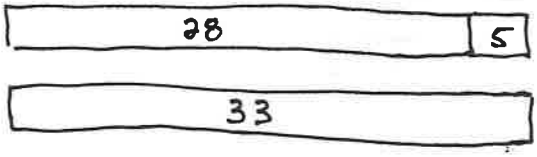
Final Answer:

Tutorial Solutions

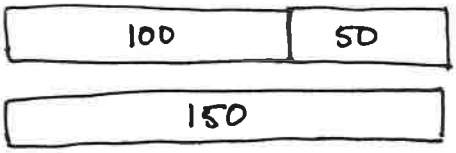
Addition page 9

3 + 5 	1 + 2 
7 + 9 	28 + 5 

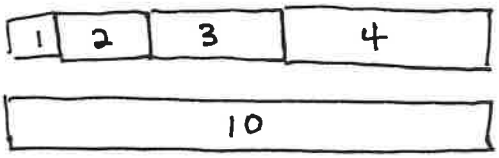
Addition page 10

3 + 5 	1 + 2 
7 + 9 	28 + 5 

100 + 50



1 + 2 + 3 + 4



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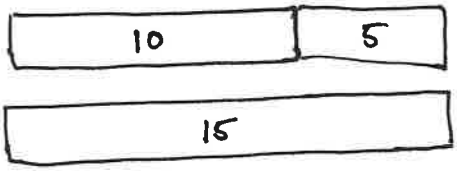

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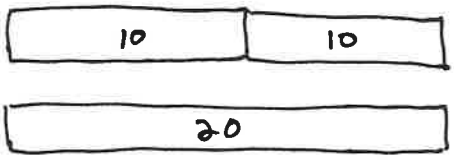
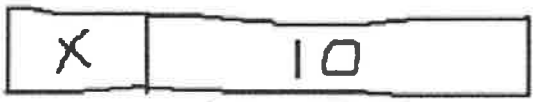
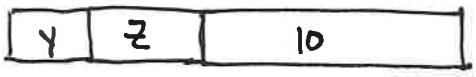


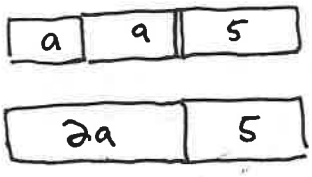
x + 10



y + z + 10

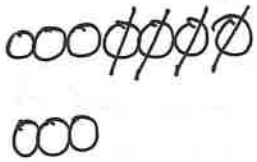
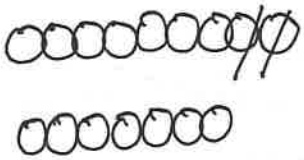
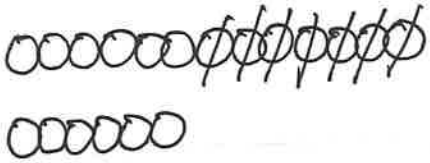


a + a + 5

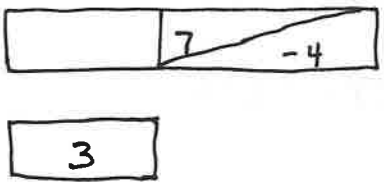
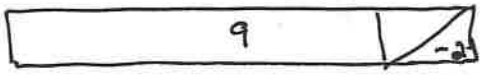
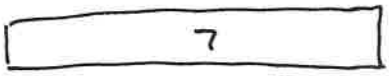
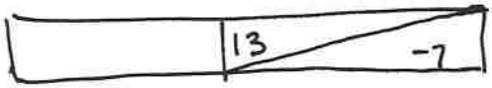
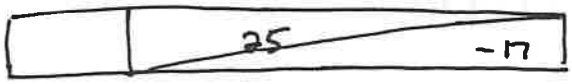
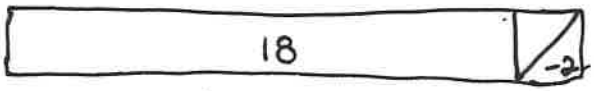
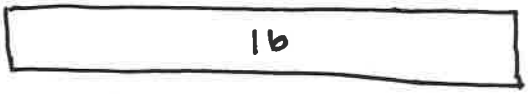
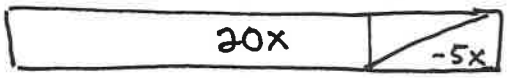
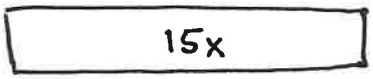
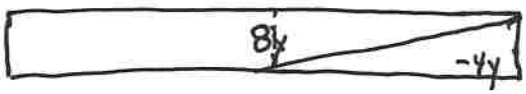
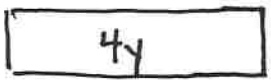


b + 1 + 2

Subtraction page 12

7 - 4 	9 - 2 
13 - 7 	25 - 17 

Subtraction page 14

7 - 4  	9 - 2  
13 - 7  	25 - 17  
18 - 2  	20x - 5x  
8y - 4y  	30a - 10a  

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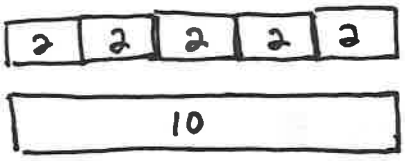

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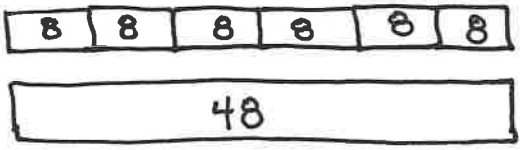

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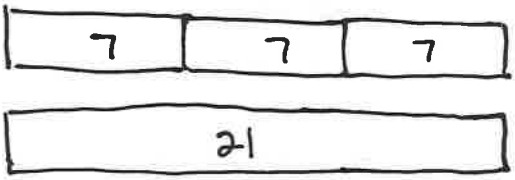

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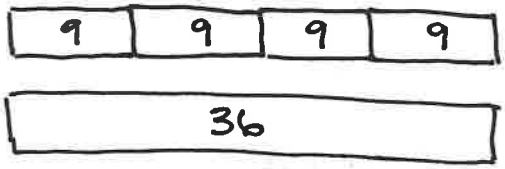

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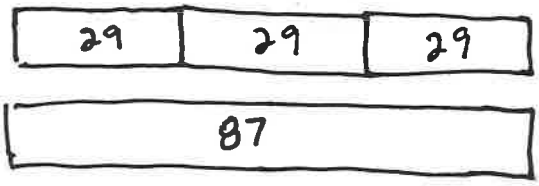

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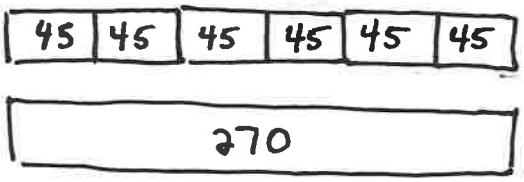

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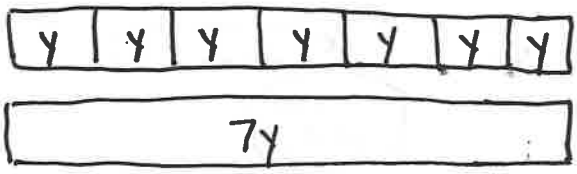

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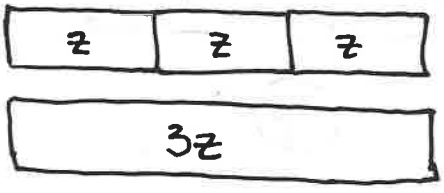
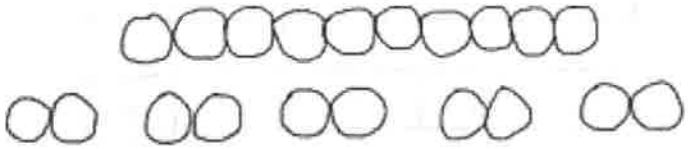
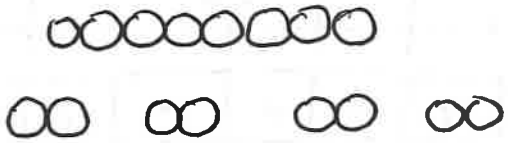
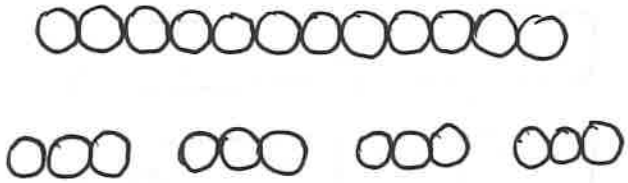
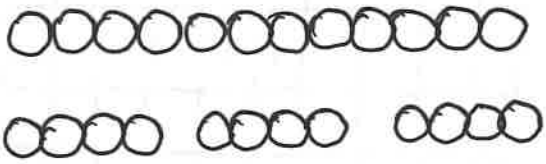
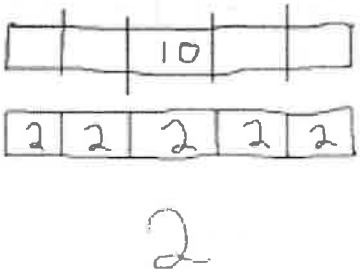
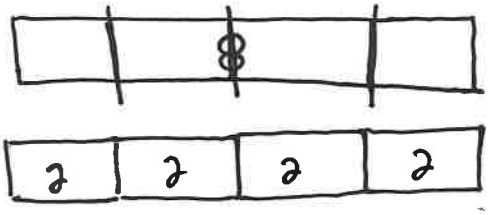
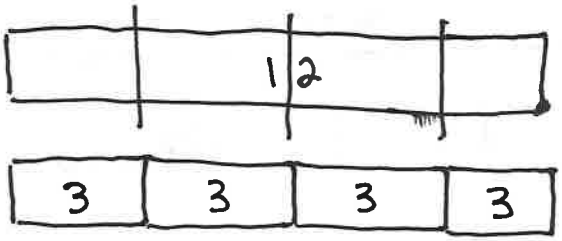
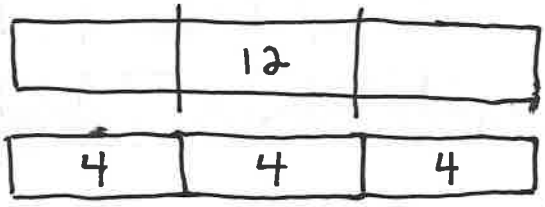


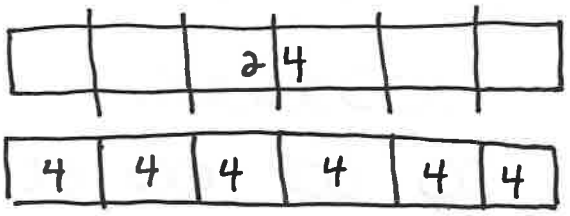
Division page 17

10 ÷ 5 	8 ÷ 4 
12 ÷ 4 	12 ÷ 3 

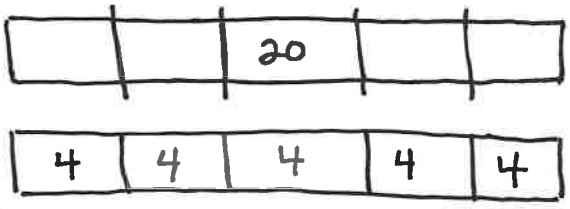
Division page 18

10 ÷ 5  2	8 ÷ 4 
12 ÷ 4 	12 ÷ 3 

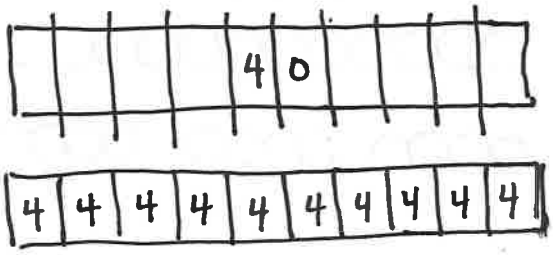
24 ÷ 6



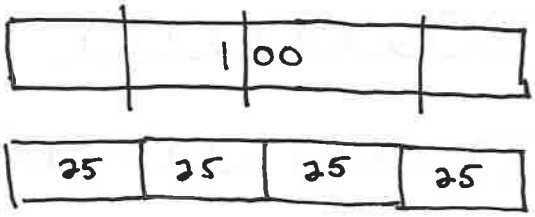
20 ÷ 5



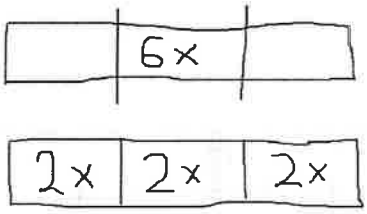
40 ÷ 10



100 ÷ 4

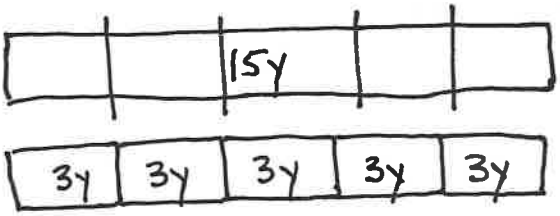


6x ÷ 3

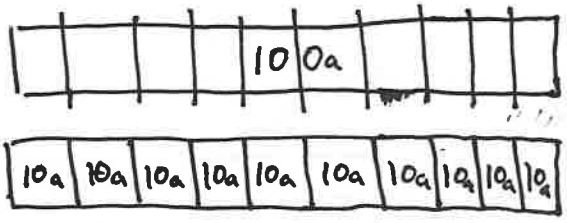


2x

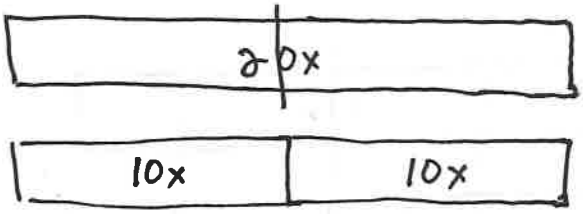
15y ÷ 5



100a ÷ 10

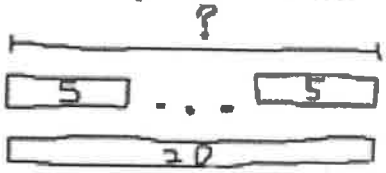


20x ÷ 2



20 ÷ 5

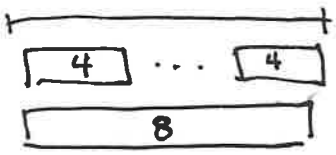
5 × ? = 20



4

8 ÷ 4

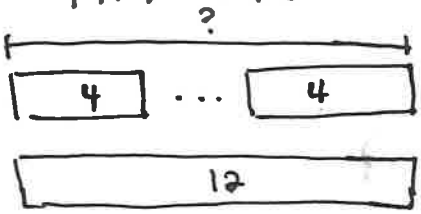
4 × ? = 8



2

12 ÷ 4

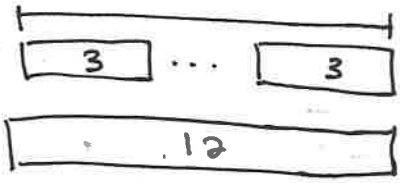
4 × ? = 12



3

12 ÷ 3

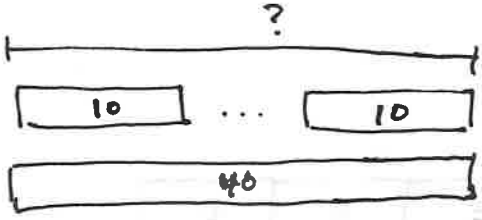
3 × ? = 12



4

40 ÷ 10

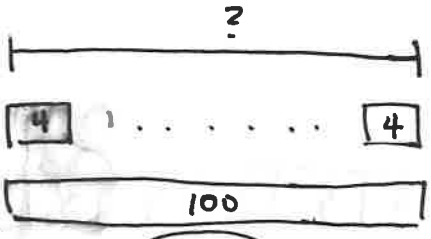
10 × ? = 40



4

100 ÷ 4

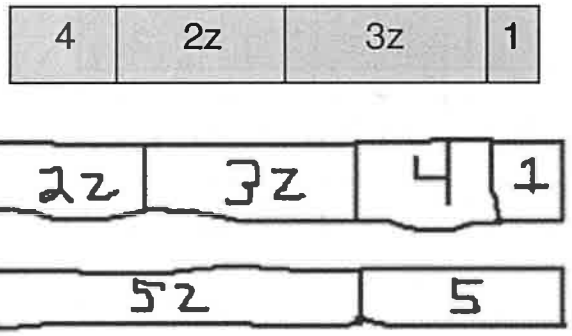
4 × ? = 100



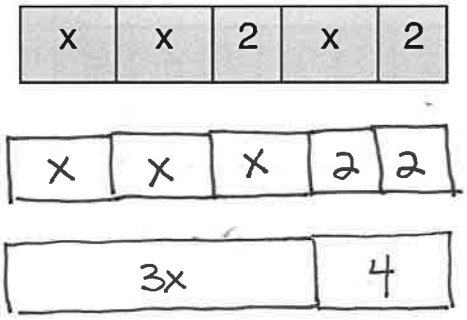
25

Simplifying page 22

4 + 2z + 3z + 1

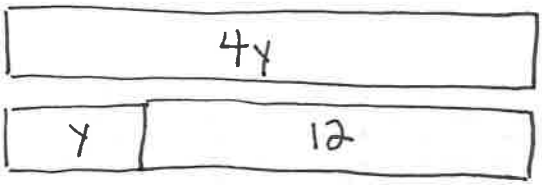


x + x + 2 + x + 2

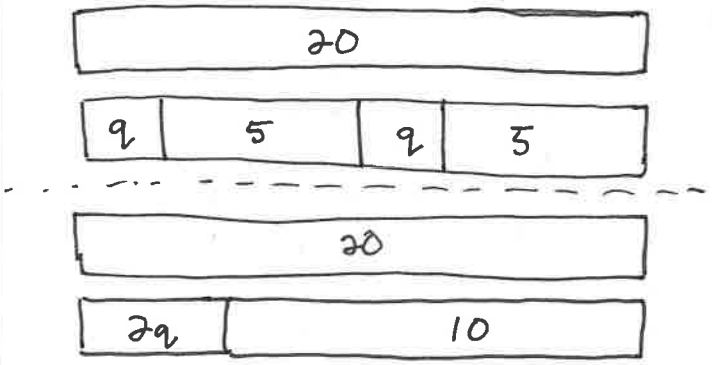


Simplifying page 24

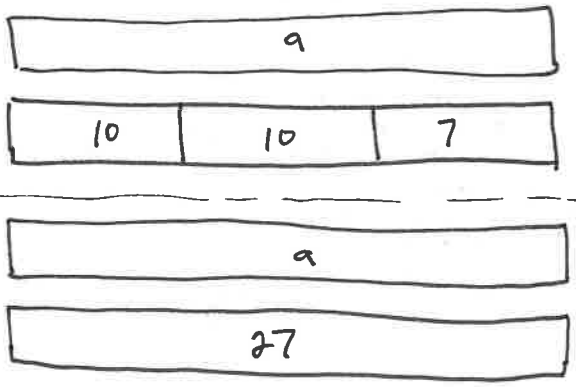
4y = y +12



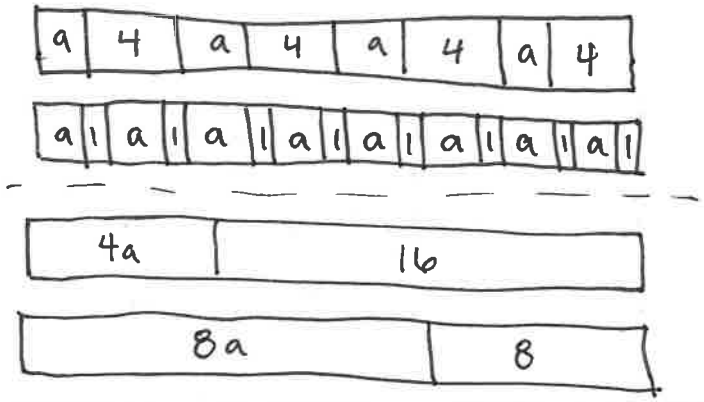
20 = 2(q+5)



a = 2(10)+7

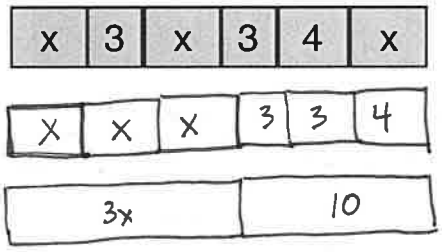


4(a+4) = 8(a+1)

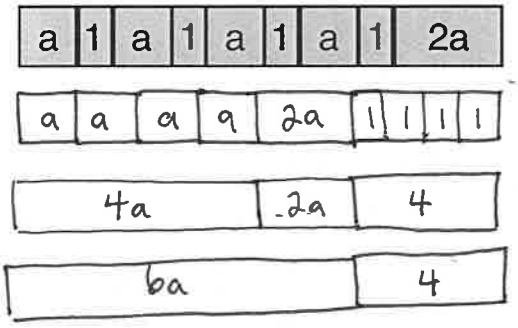


Simplifying page 23

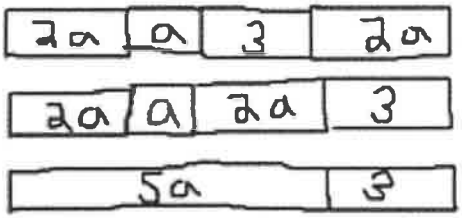
$2(x + 3) + 4 + x$



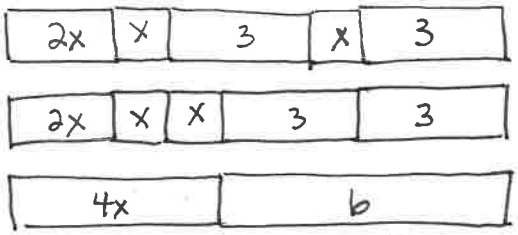
$4(a + 1) + 2a$



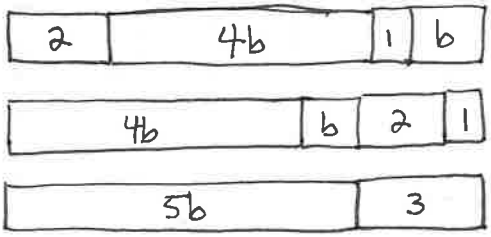
$2a + a + 3 + 2a$



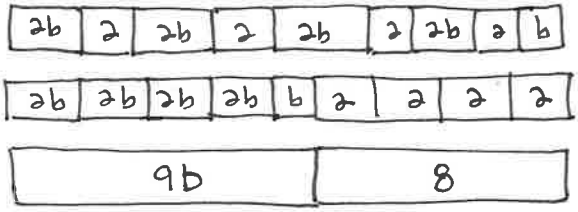
$2x + 2(x + 3)$



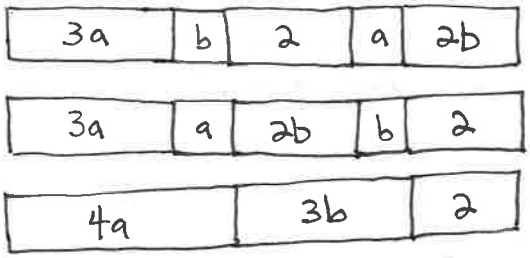
$2 + 4b + 1 + b$



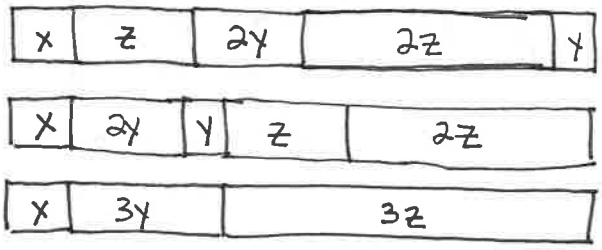
$4(2b + 2) + b$



$3a + b + 2 + a + 2b$

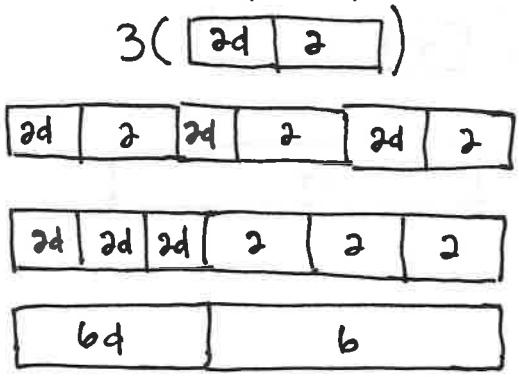


$x + z + 2y + 2z + y$

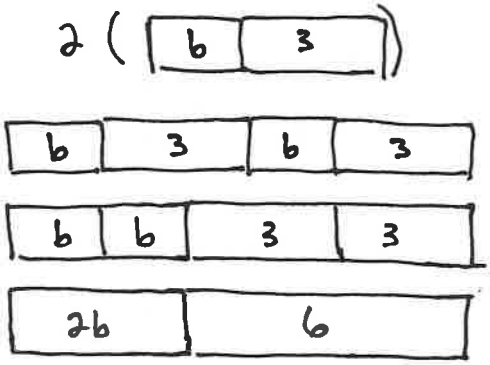


Distributive Property pages 36-37

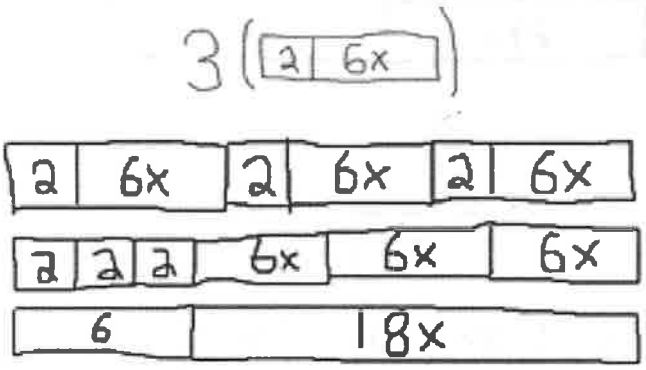
3(2+2d)



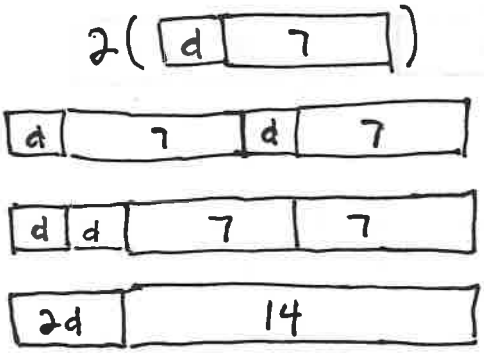
2(b+3)



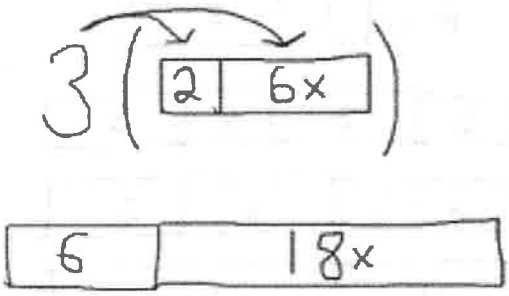
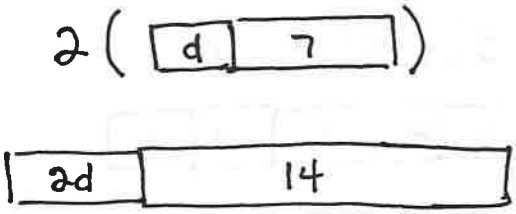
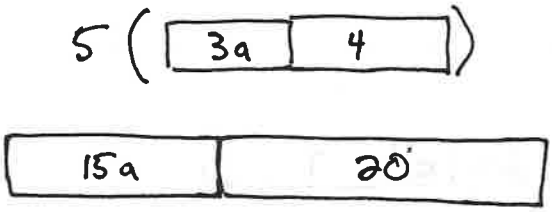
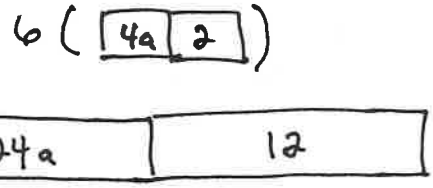
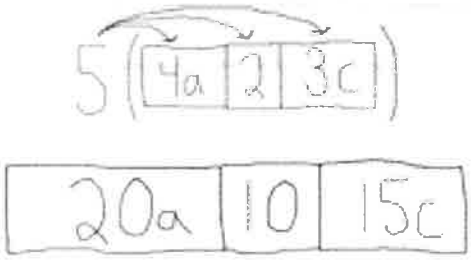
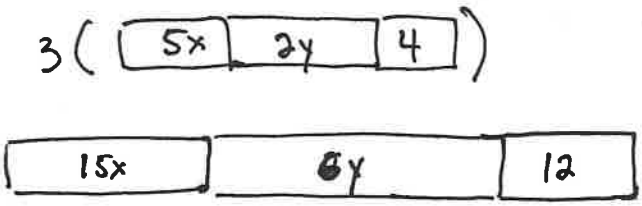
3(2+6x)



2(7+d)

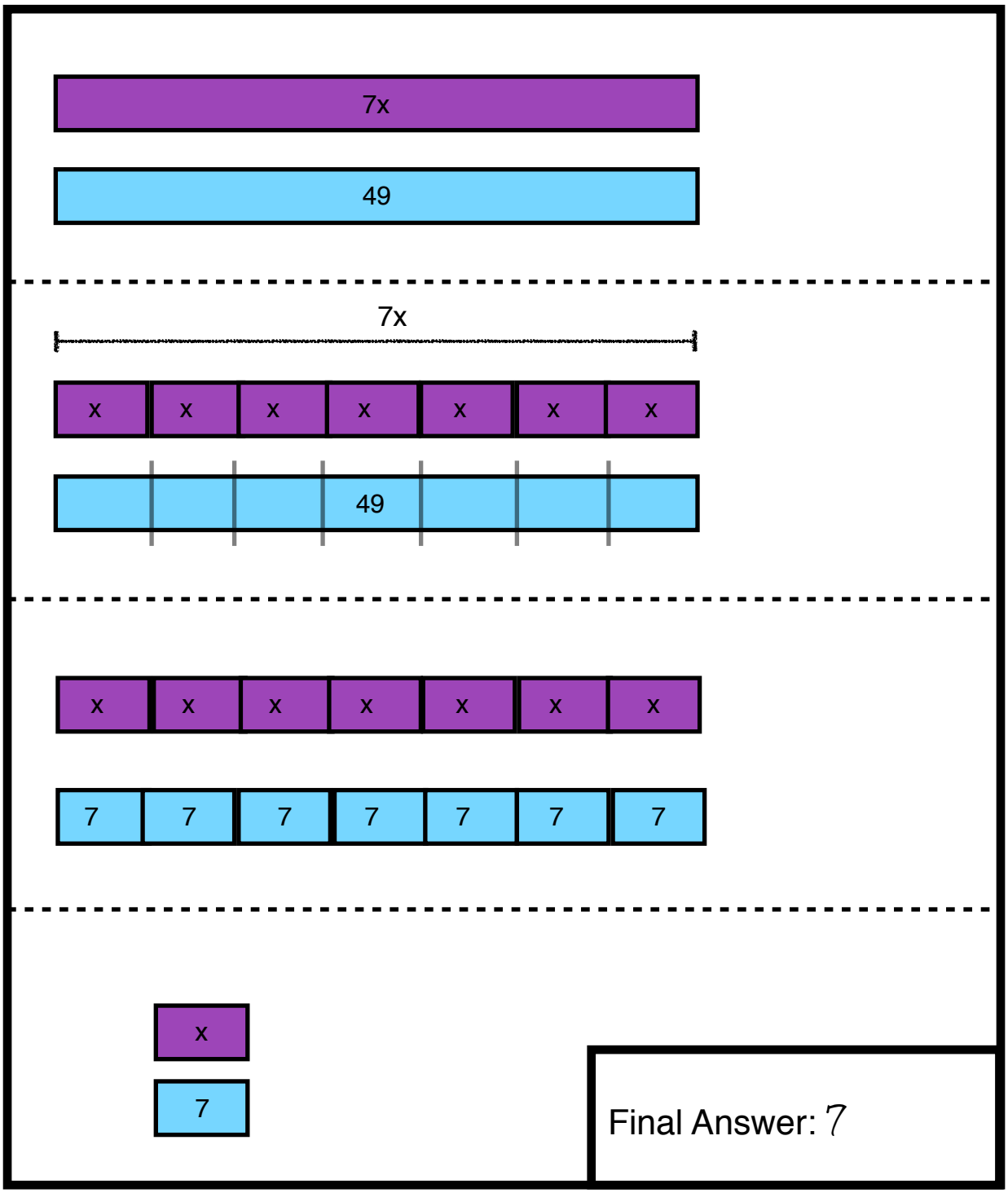


Distributive Property page 37

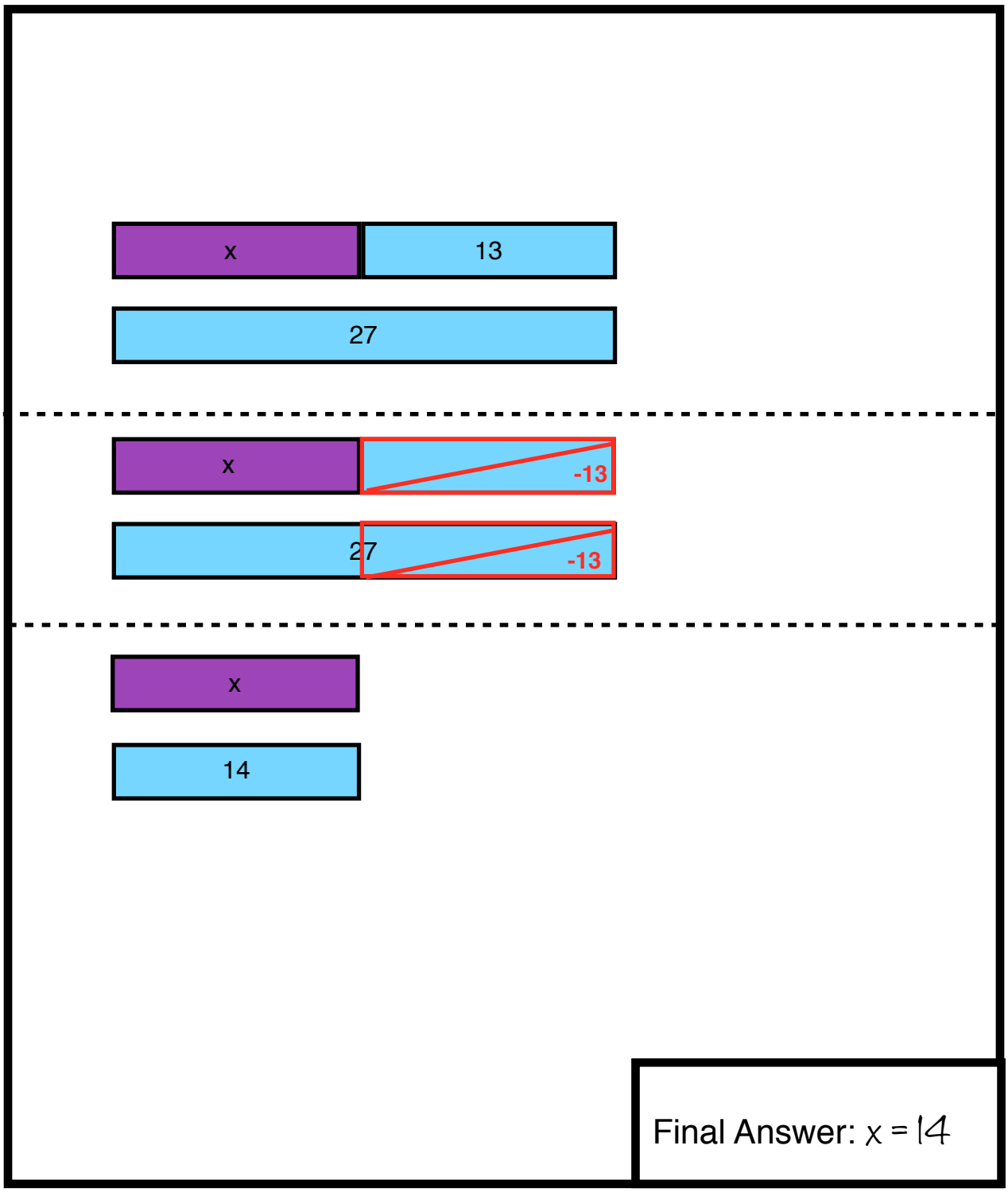
3(2+6x)  	2(7+d) 
5(4+3a)  	6(4a+2) 
5(4a+2+3c)  	3(5x+2y+4) 

Solutions

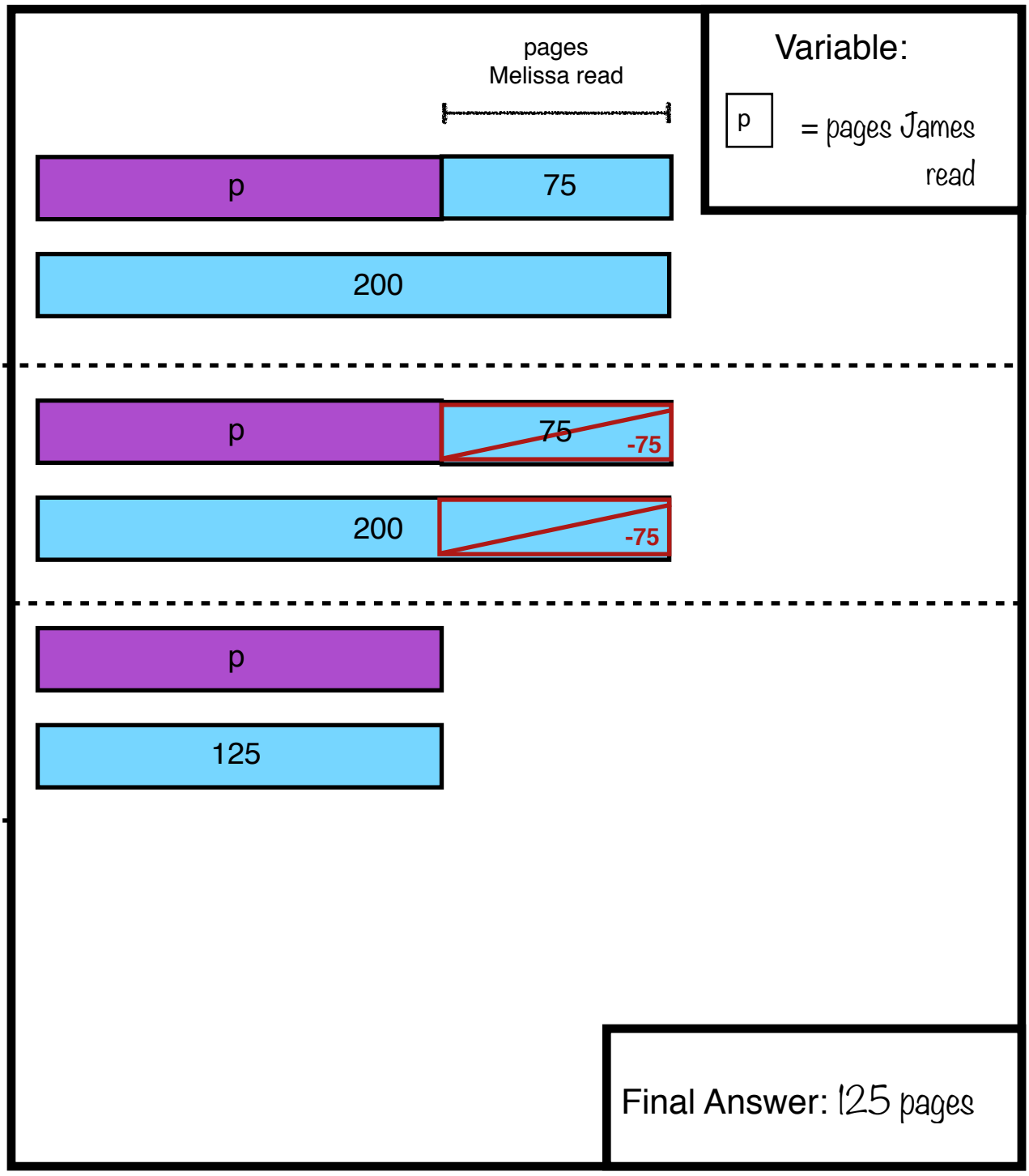
Problem: 7x=49



Problem: x + 13 = 27



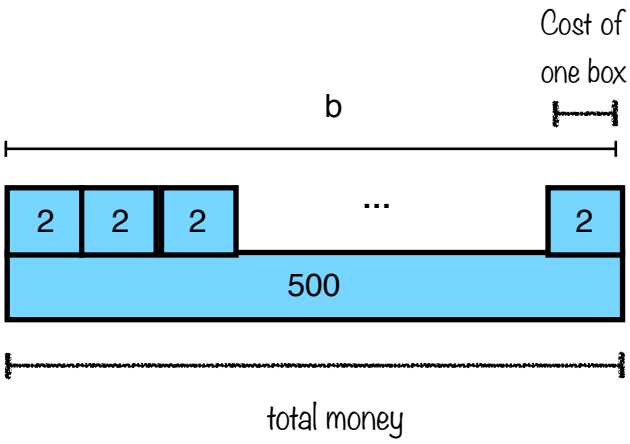
Problem: James and Melissa have a reading assignment to complete. Melissa has read 75 pages. Together they have read a total of 200 pages. How many pages did James read?



Problem: Jake's class is selling boxes of candy for a fundraiser. Their goal is to raise 500 dollars. If they make $2 for each box of candy they sell, how many boxes do they need to sell to reach their goal?

Variable:

b = number of boxes

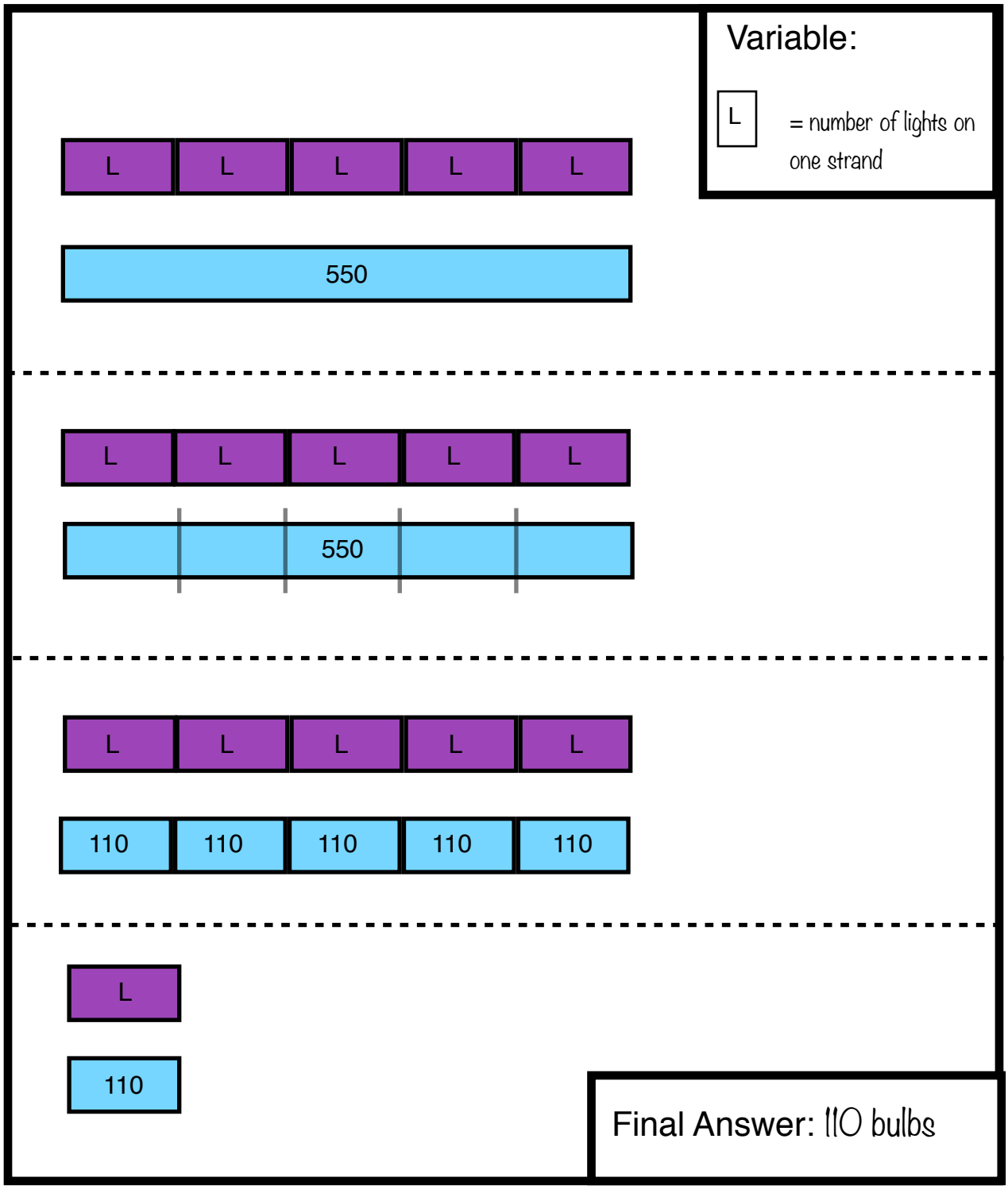


From the model we can see that we now have to divide 500 by 2 in order to find the answer!

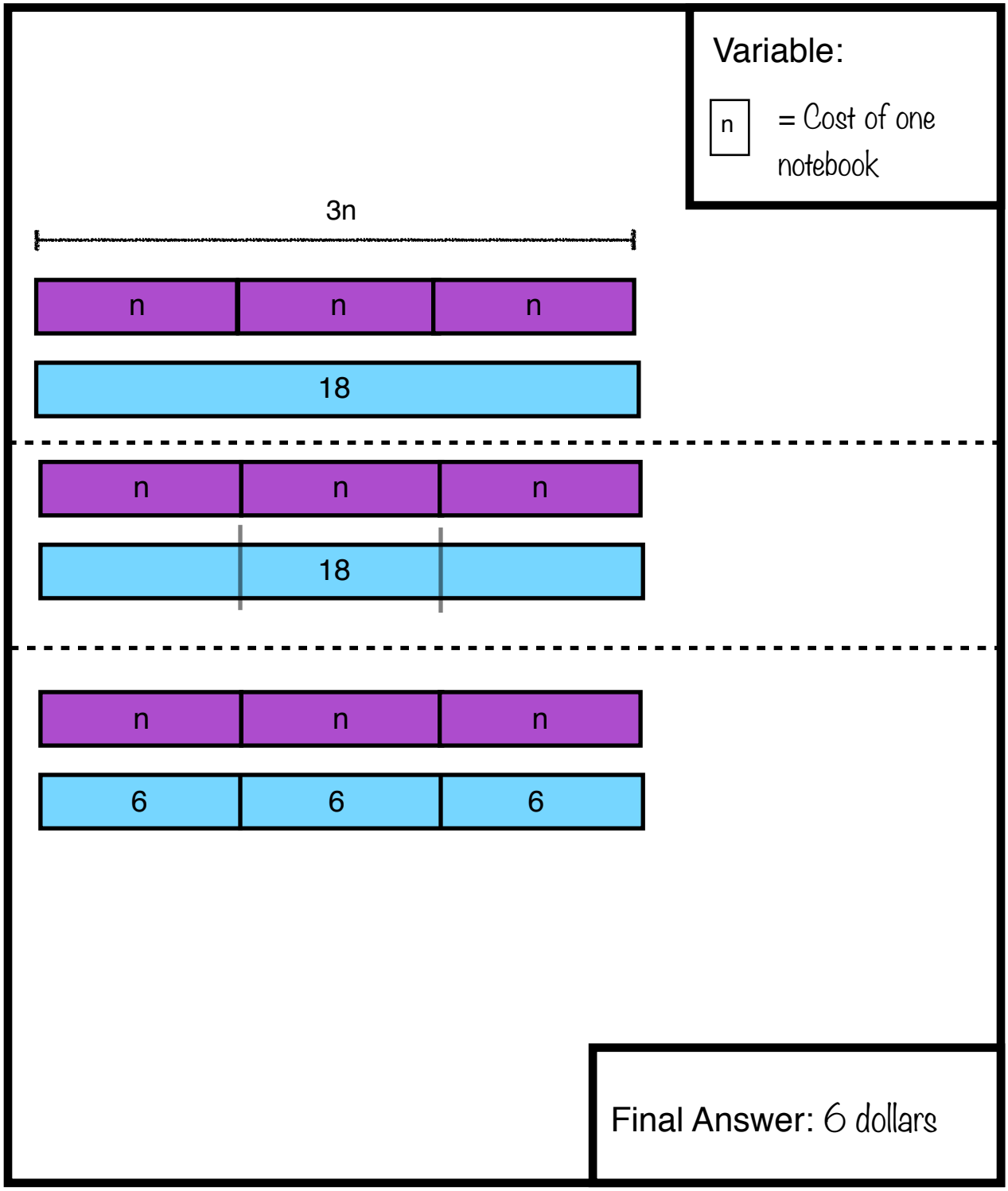
$500 \div 2 = 250$

Final Answer: 250 boxes

Problem: Jimmy loves Christmas lights! He knows he has 550 total light bulbs and he has 5 strands of lights with the same number of bulbs on each. How many lights are on each strand that Jimmy has?



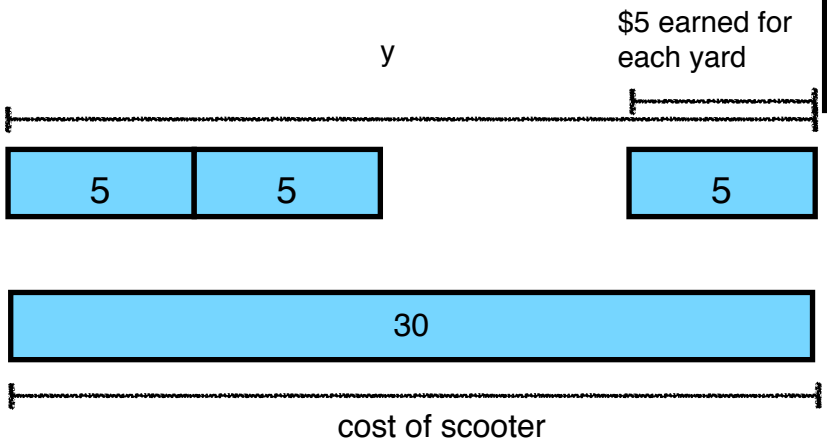
Problem: Mark bought three notebooks. In total he spent $18. How much did he spend on each notebook?



Problem: Alicia wants to buy a new scooter for $30. She knows she can earn $5 each time she mows a neighbor's yard. How many yards will she have to mow to afford the scooter?

Variable:

y = number of yards



Looking at this model, we can see that we must divide 30 by 5 in order to find the answer:

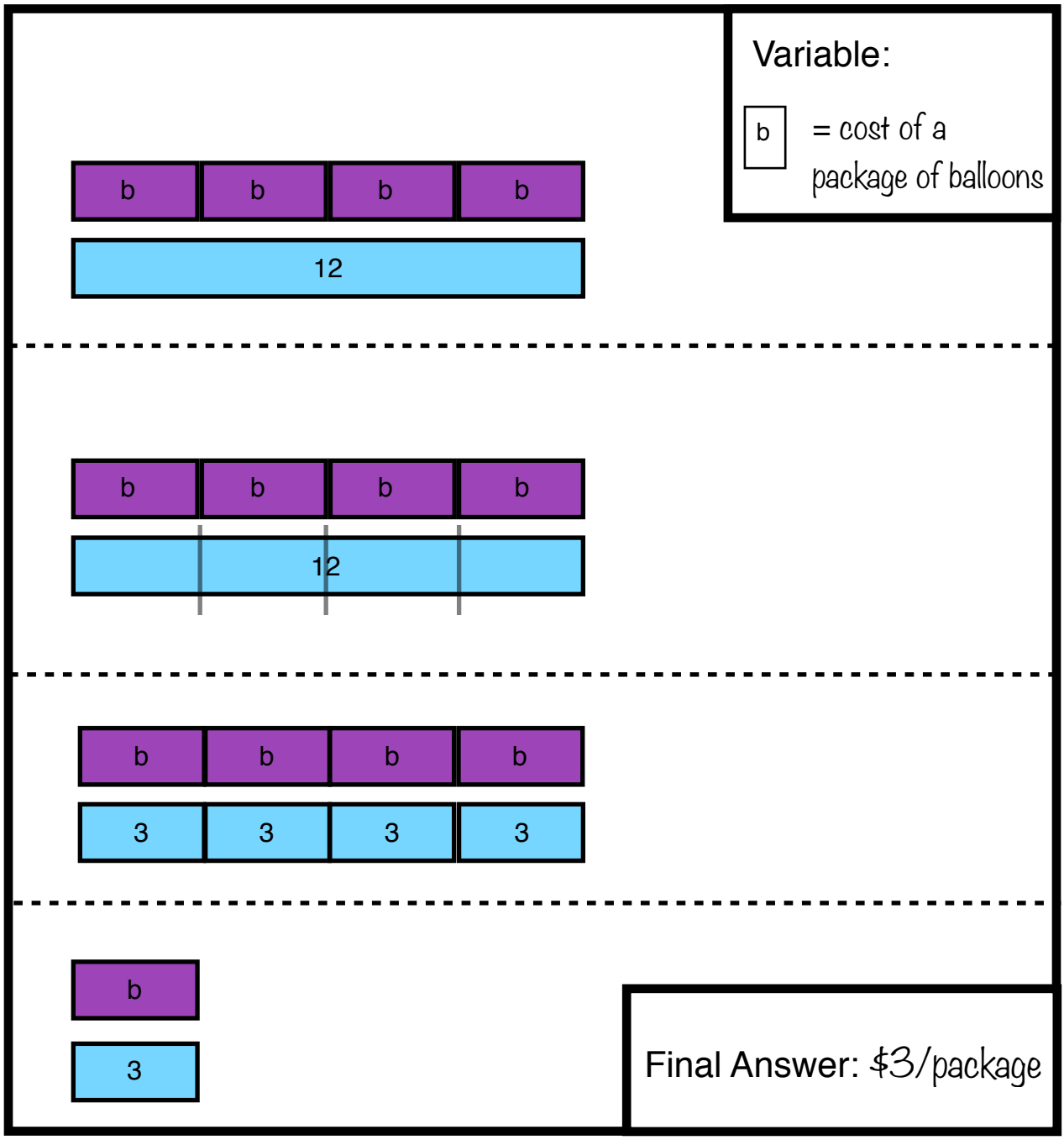
$30 \div 5 = 6$

Let's check:
$5 x 6 yards = $30

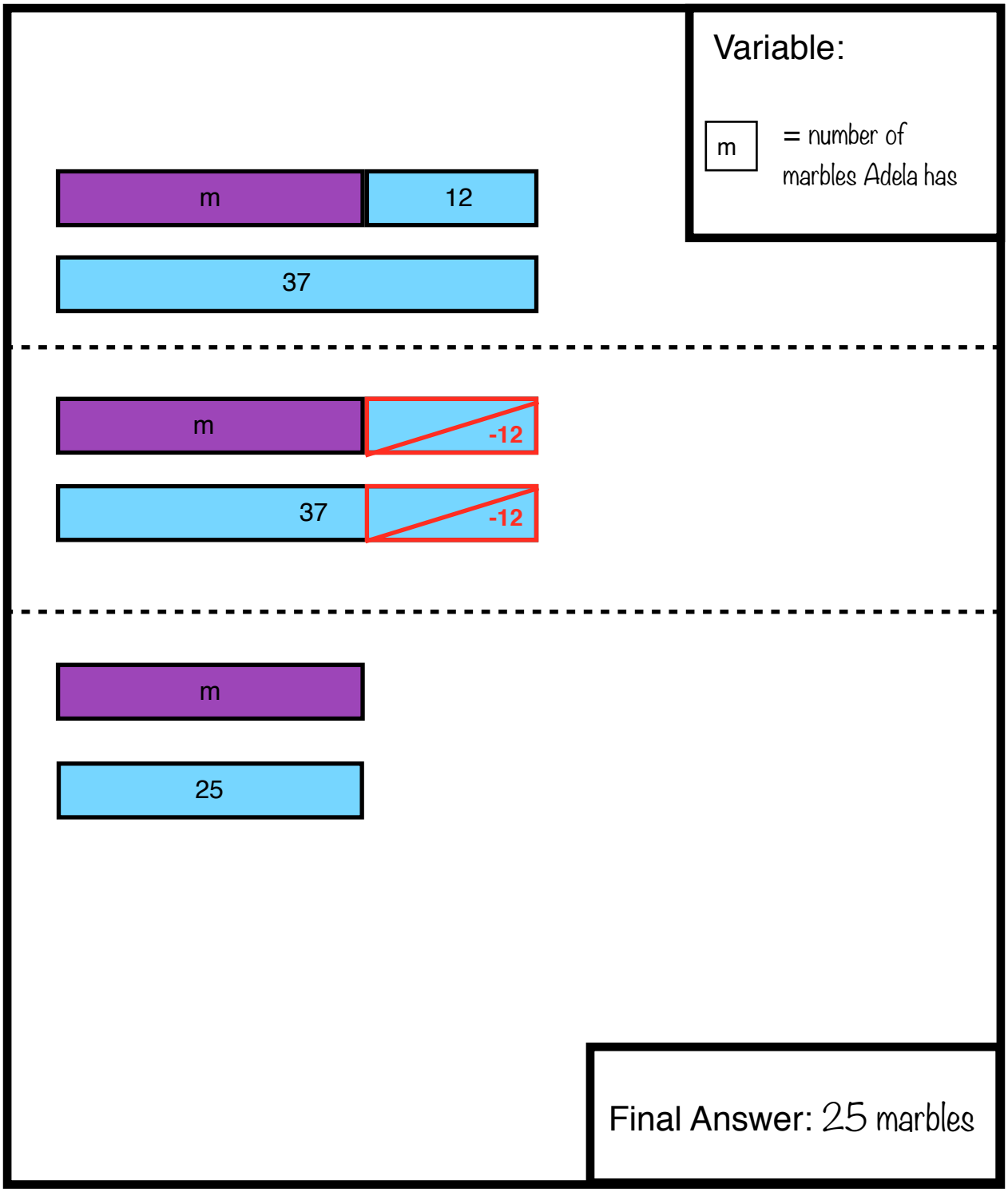
She has enough for the scooter!

Final Answer: 6 yards

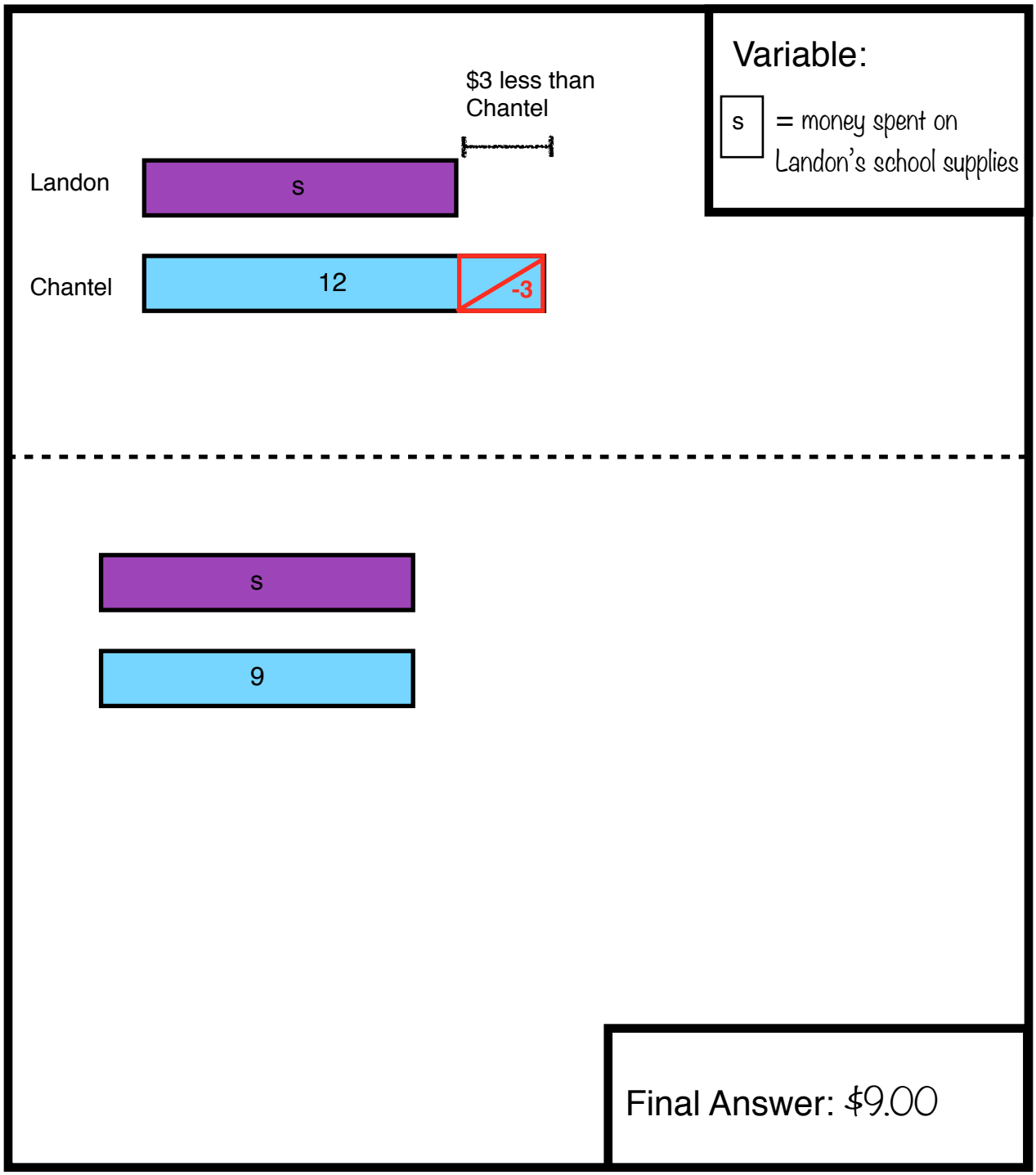
Problem: Abdul and Maria are planning a surprise party for their friend. Together they have $12 to spend on balloons. If they buy 4 packages of balloons and spend all their money, how much does a package of balloons cost?



Problem: Izzy collects marbles. She currently has 12. Her older sister, Adela, tells her that if they combined their collections, they would have 37 marbles total. How many marbles does Adela have?



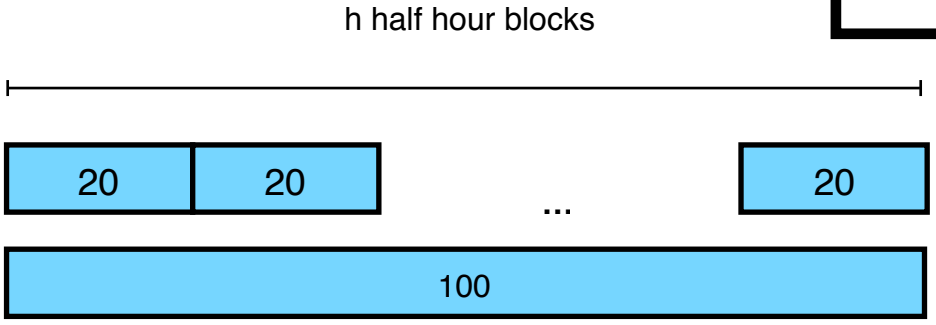
Problem: Landon and Chantel are buying supplies for school. Landon spends $3 less than Chantel. Chantel spends $12. How much did Landon spend on school supplies?



Problem: Christina has to finish a book before her class tomorrow. She has 100 pages left. If Christina knows that she can read twenty pages in a half an hour, how long (in hours) will it take her to finish her reading?

Variable:

h = number of 1/2-hour blocks



Looking at the model we see we need to divide 100 by 20 to find how many half-hour blocks she needs to finish.

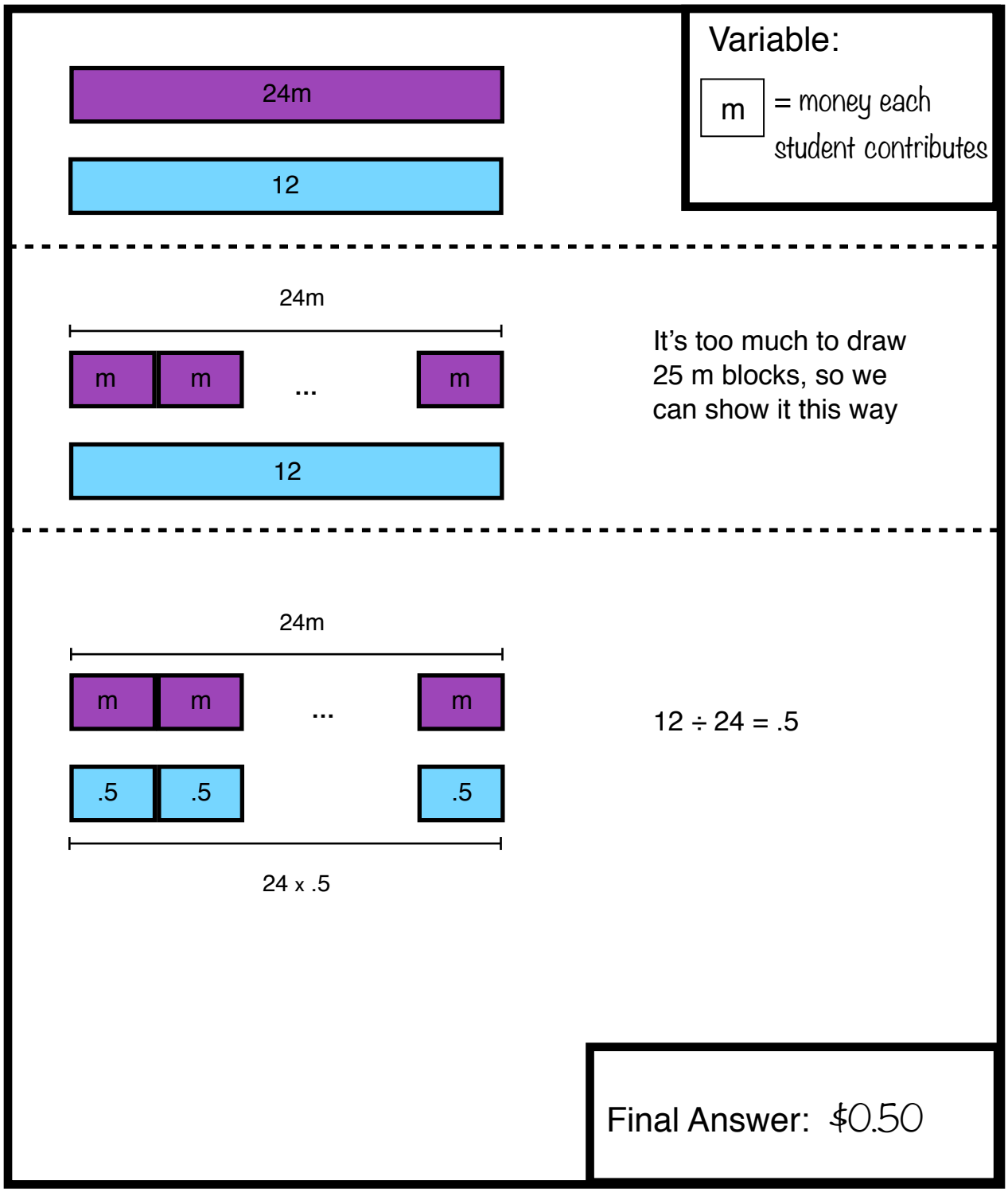
$100 \div 20 = 5$

Let's check,
5 half-hour blocks * 20 pages = 100 pages.

But we want the answer in hours, so
5 ½-hour blocks = 2.5 hours

Final Answer: 2.5 hours

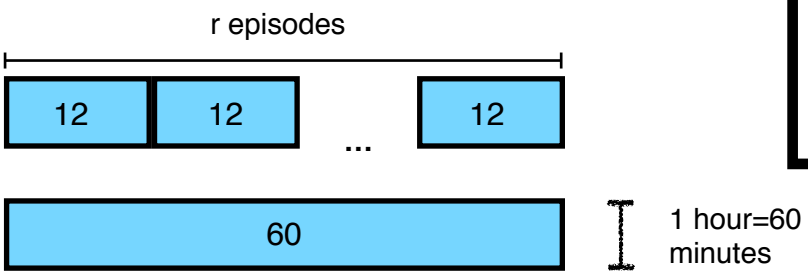
Problem: Mrs. Dunn's class decides that they want to buy her an end of the year gift. There are 24 students in the class. They want to buy Mrs. Dunn a picture frame that costs $12, how much will each student contribute to split the cost evenly?



Problem: Ray has to go to soccer practice in an hour. He wants to watch some of his favorite cartoons before leaving. If each episode is twelve minutes, how many can he watch before he needs to leave?

Variable:

r = number of episodes



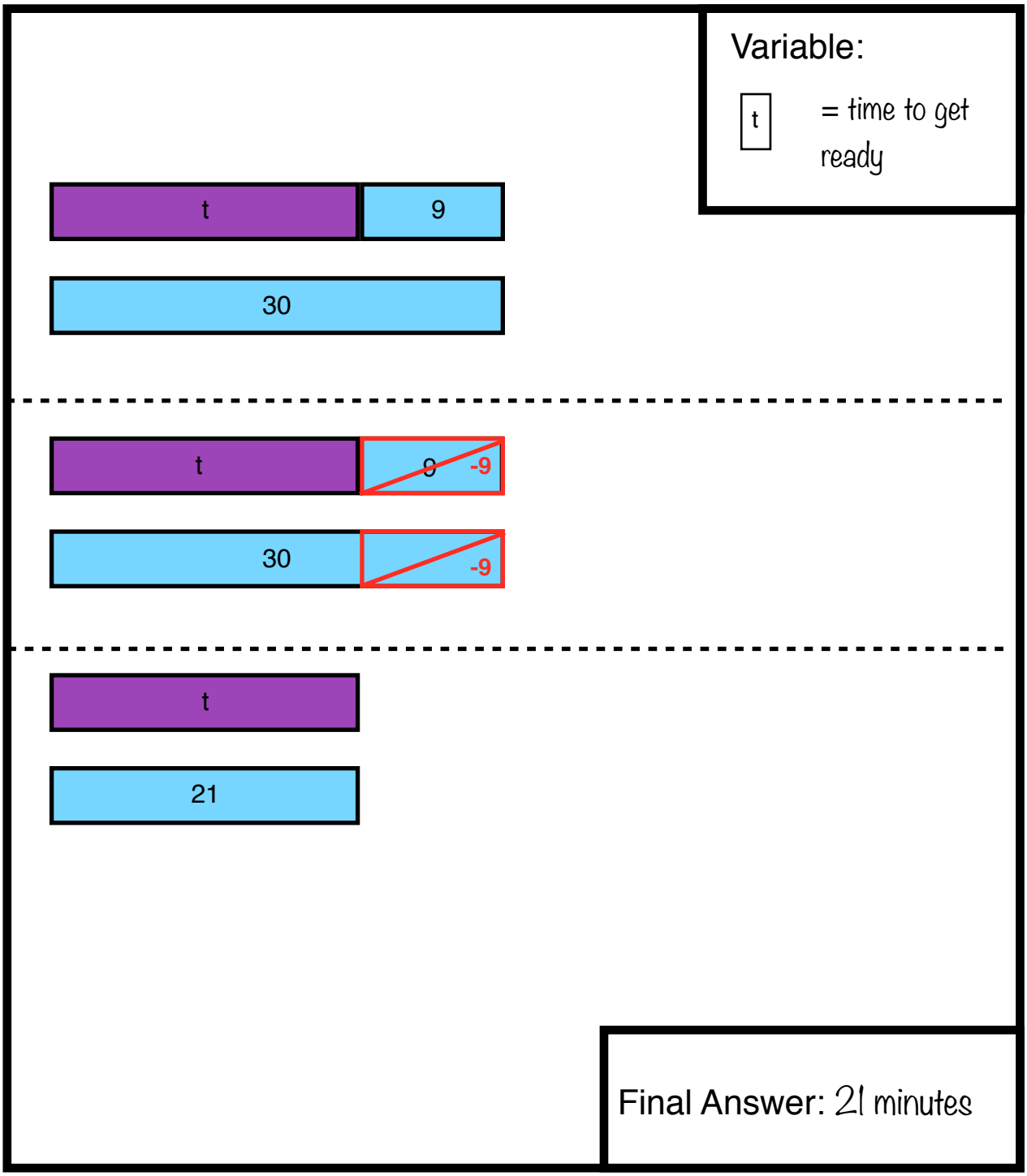
I can see from the model that I need to divide 60 by 12 to find the number of episodes

$60 \div 12 = 5$

Let's check:
5 episodes x 12 minutes per episode = 60 minutes of TV

Final Answer: 5 episodes

Problem: Felicity has 30 minutes before her bus will come to pick her up. If it takes 9 nine minutes to eat breakfast, how much more time does she have to get ready for school?



Problem: Sean's class is going on a field trip with a small participation fee. There are 10 students in his class, who all paid the same amount. All together, they paid $25.00. How much did Sean have to pay?

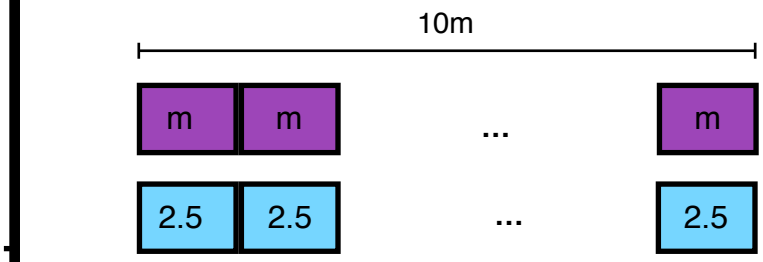
Variable:

m = money Sean pays (each student pays)

10m

25

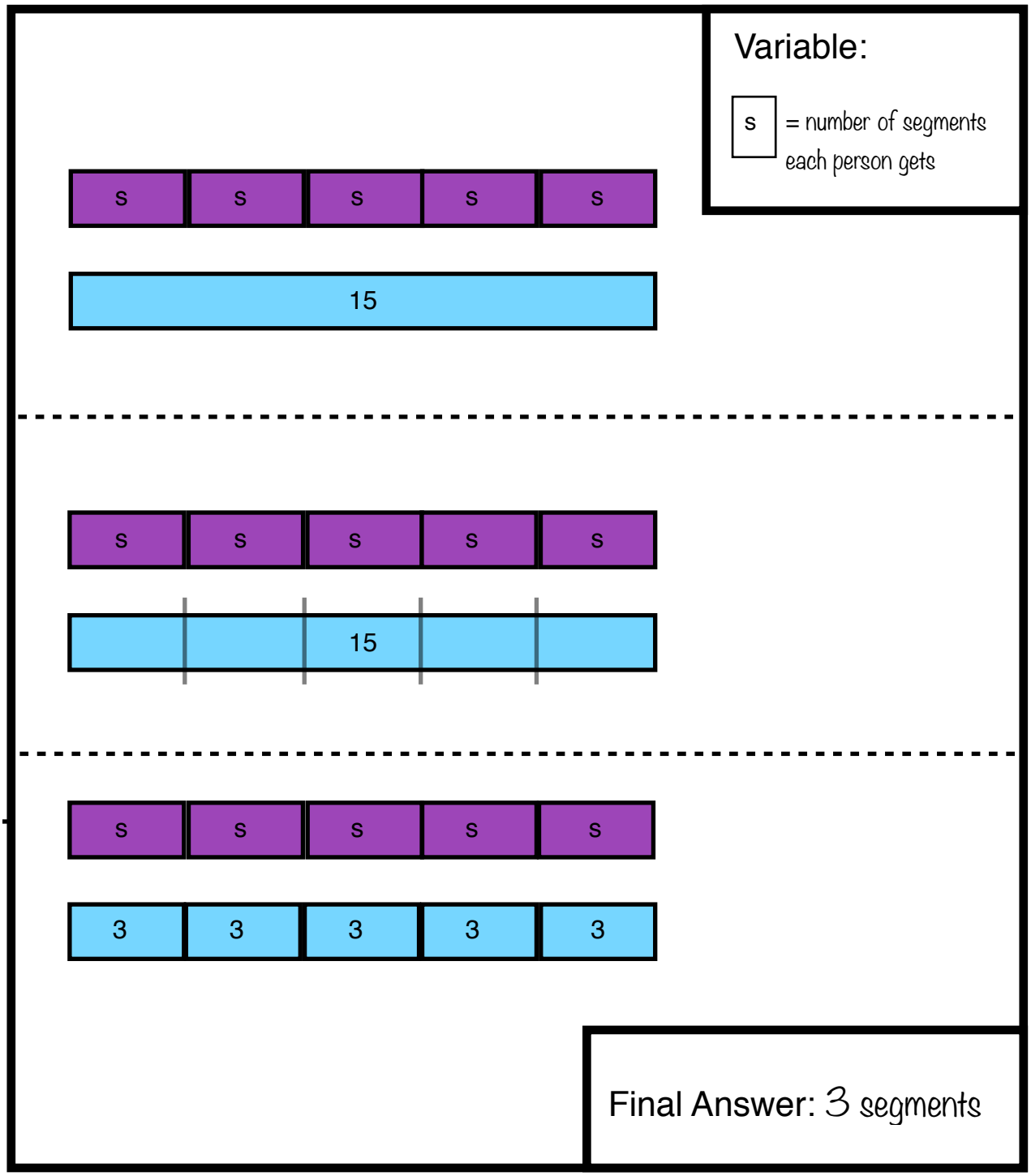
25 ÷ 10 = 2.5



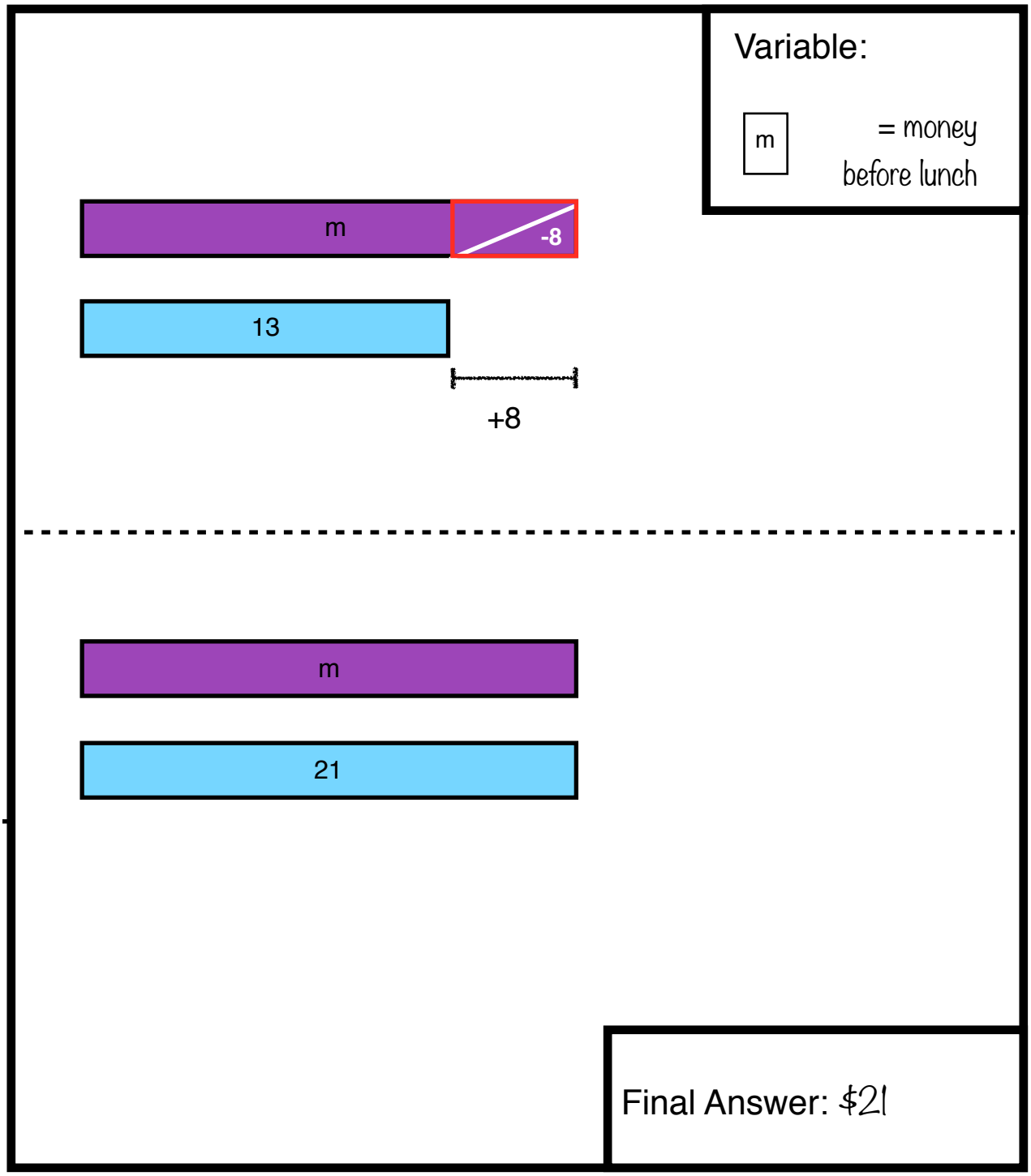
It's too much to draw 10 m blocks and 10 2.5 blocks, so we can show it this way.

Final Answer: $2.50

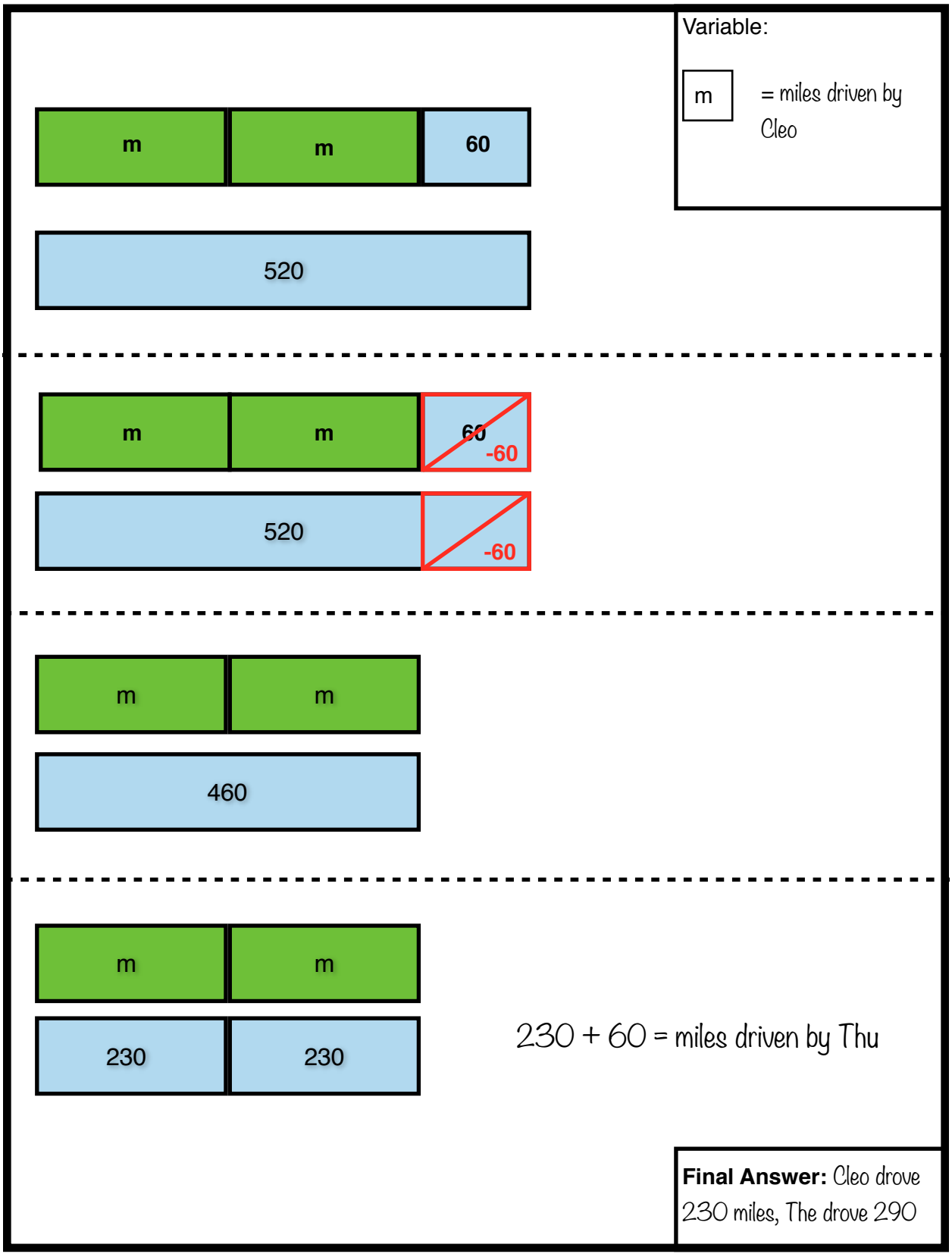
Problem: Yolanda is in charge of gathering supplies s'mores supplies for a family camping trip. There are five people in Yolanda's family and Yolanda has a package with 15 segments of chocolate. How many segments of chocolate does each person get to make s'mores?



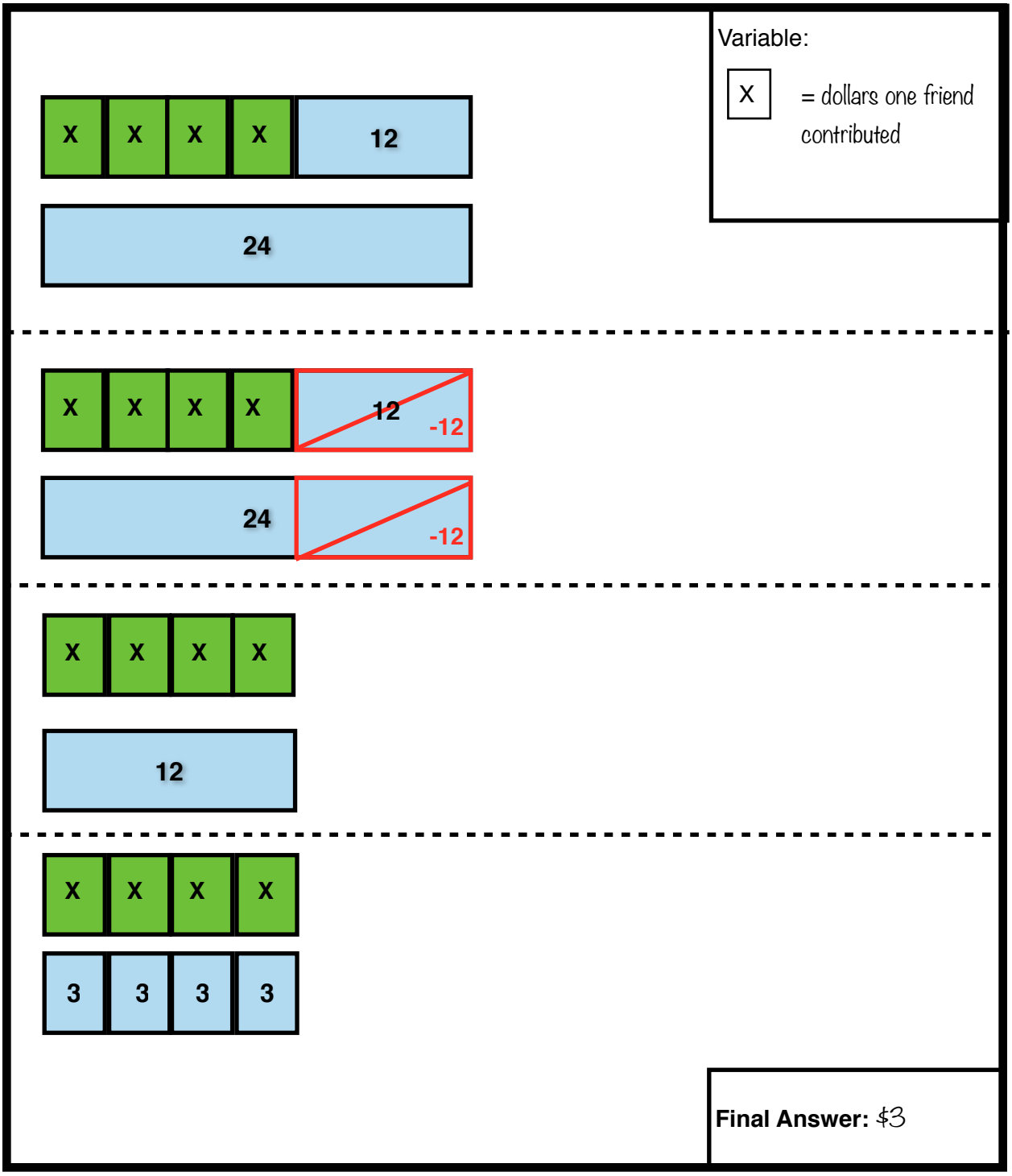
Problem: Hernando can't remember how much money he had in his wallet before lunch. He knows he spent $8 on lunch at Chipotle, and he has $13 left in his wallet now. How much money did he have before lunch?



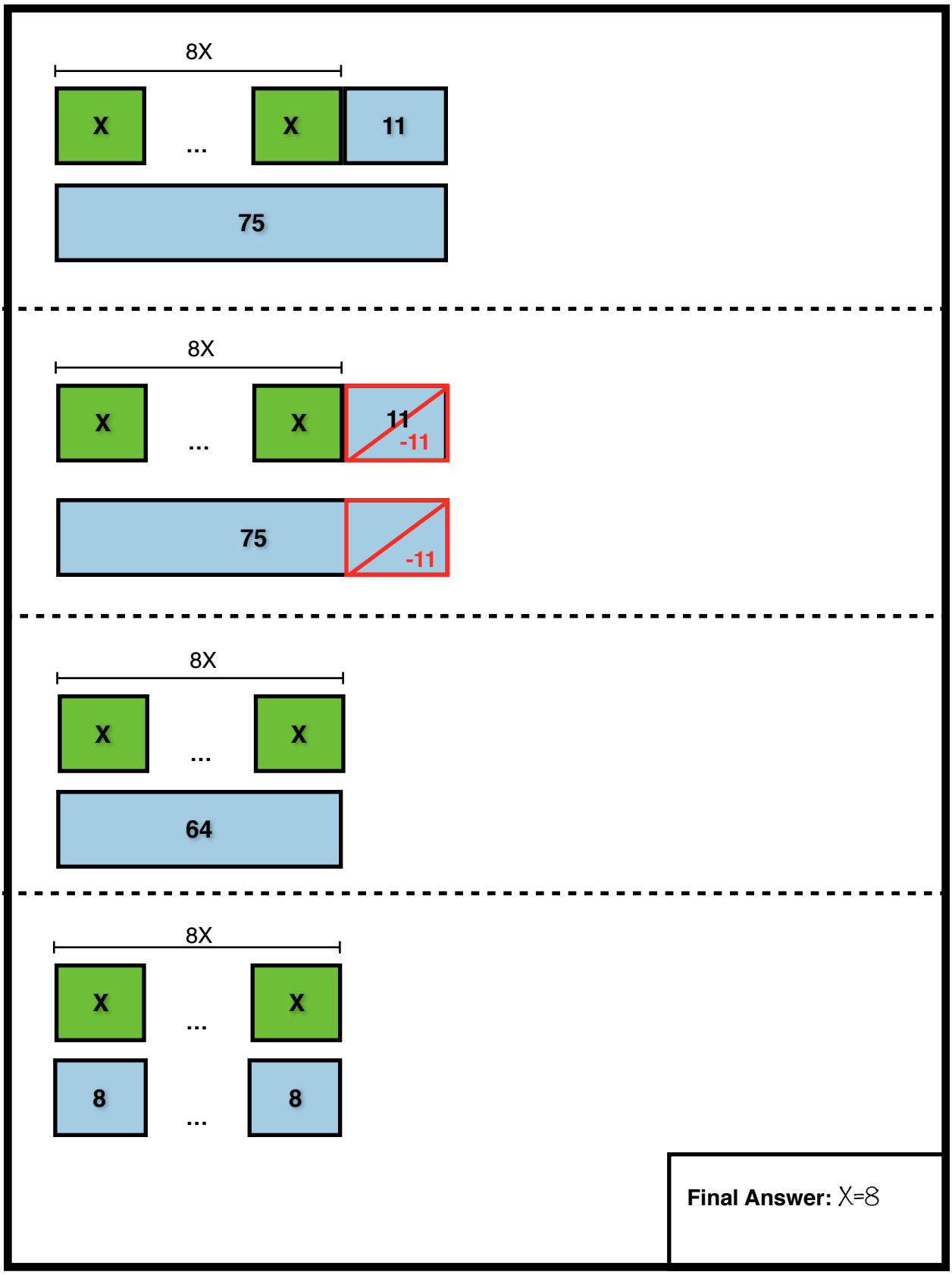
Problem: Thu and Cleo are sharing the driving on a 520 mile trip. If Thu drives 60 miles more than Cleo, how far did each of them drive?



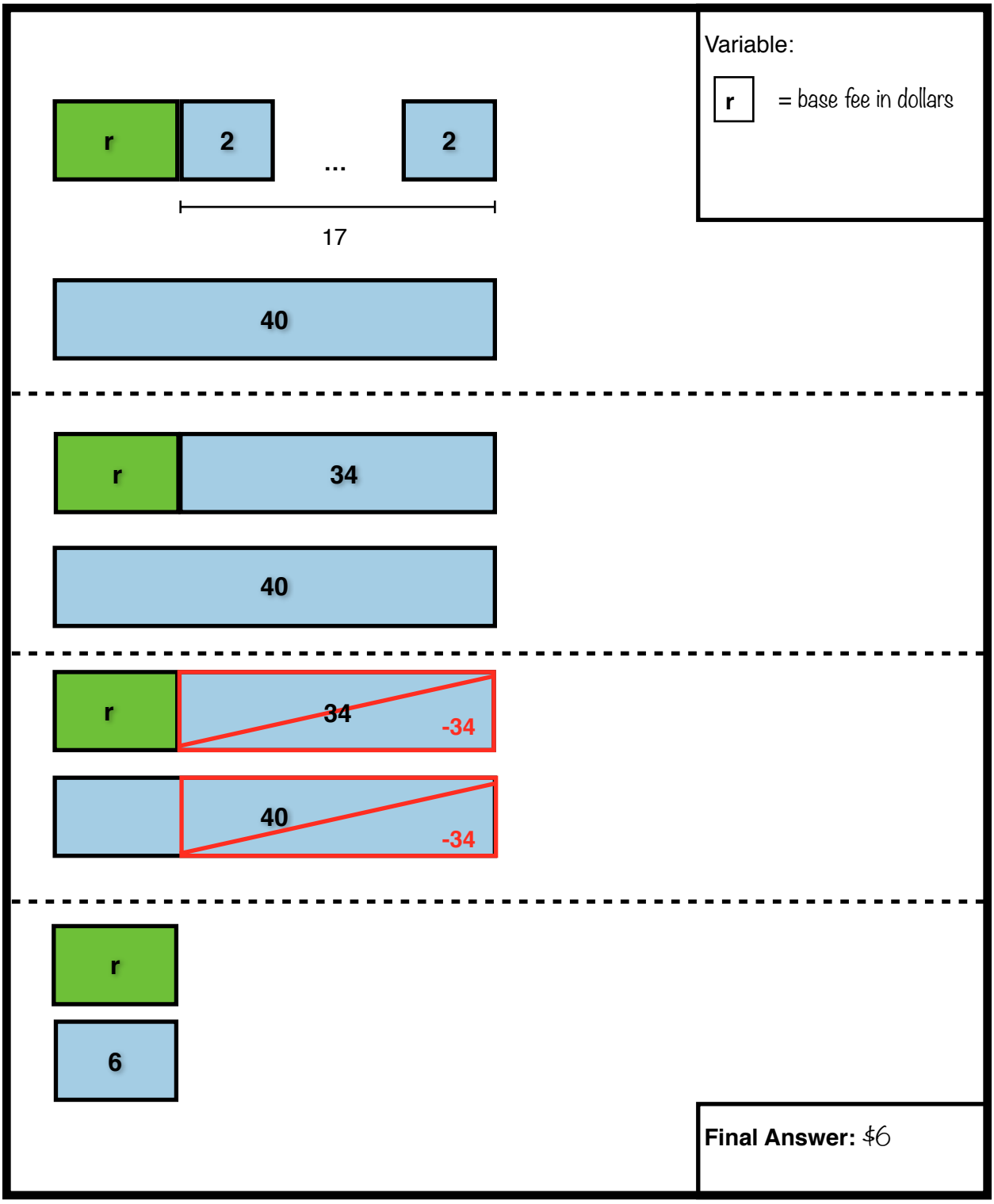
Problem: Linnea is planning on cooking dinner with three of her friends. Linnea's mom agrees to contribute $12 towards the cost of ingredients, and Linnea and each of her friends agree to split remaining cost equally. If the ingredients cost $24, how much will Linnea end up spending?



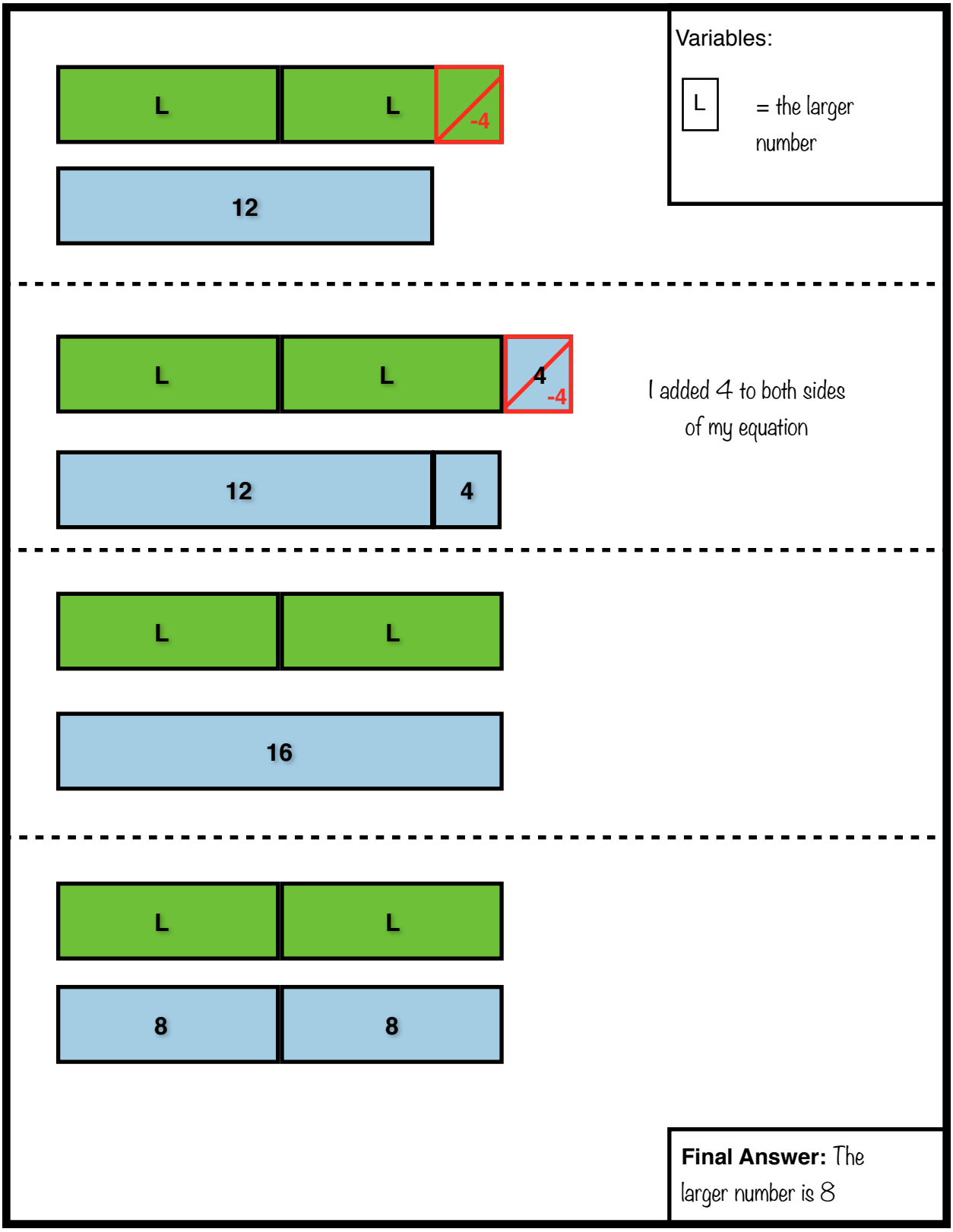
Problem: 75 = 8x + 11



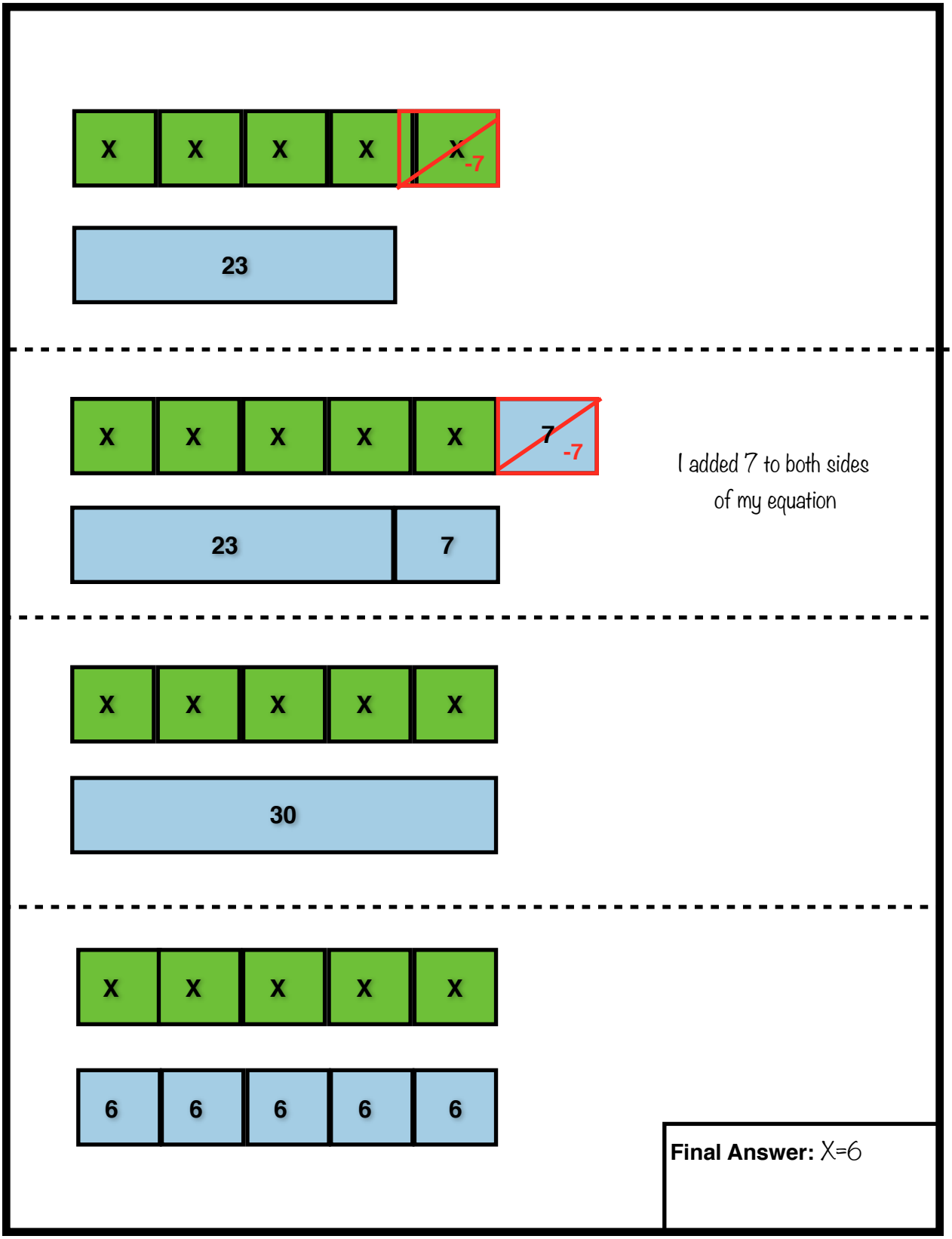
Problem: You ride a taxi for 17 minutes, and you know that they charge a base fee (an amount charged before any minutes have gone by) and $2 per minute. After the trip, $40 dollars total is charged. How much was the base fee?



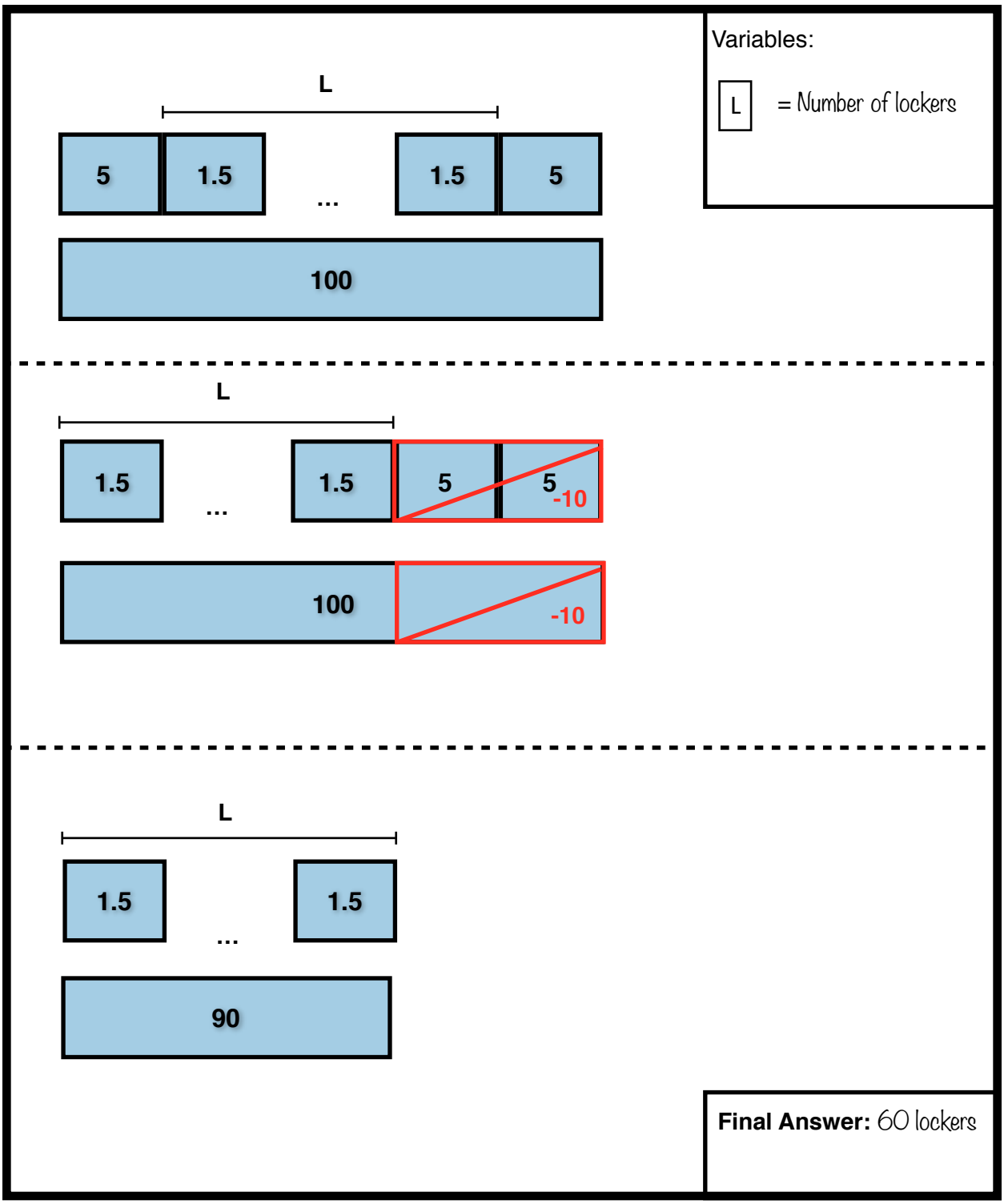
Problem: The sum of two numbers is 12. One number is 4 more than the other. What is the value of the larger number?



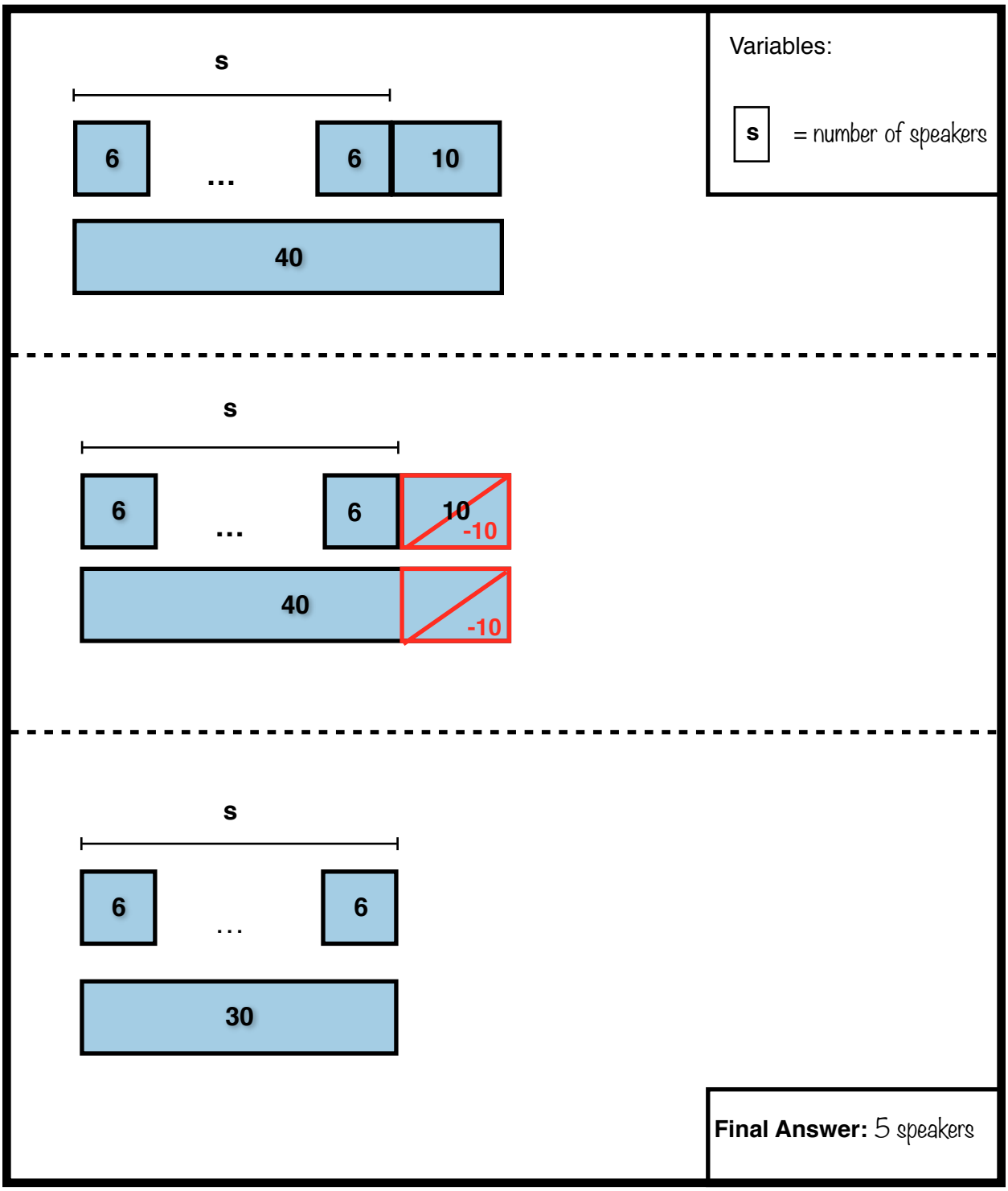
Problem: 23 = 5x - 7



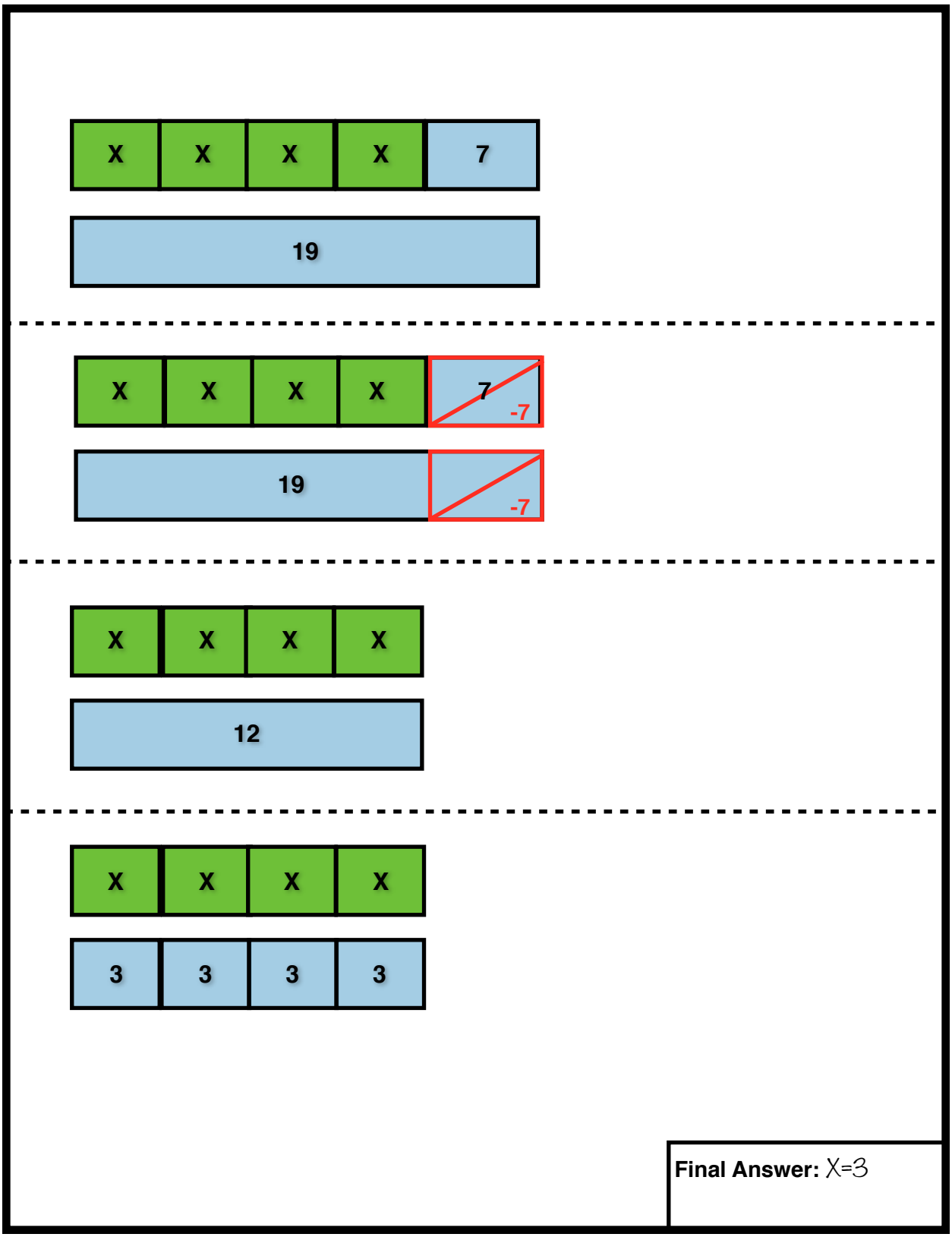
Problem: The school is putting in a new row of lockers! You have 100 feet of space, and each locker is 1 ½ feet wide. There needs to be 5 feet of extra space at each end of the row. How many lockers can you put in?



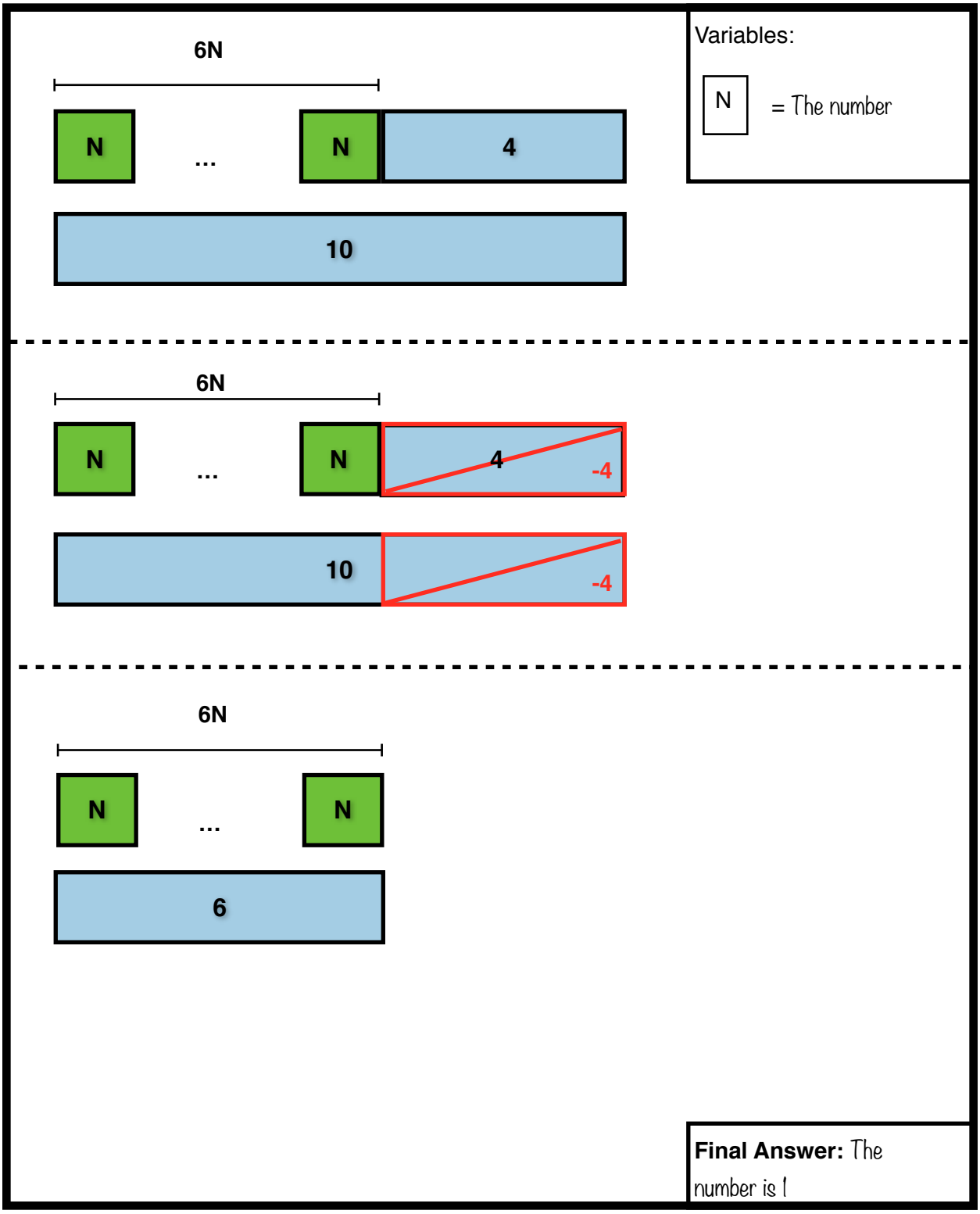
Problem: You are at school assembly and getting bored. You know the next class is supposed to start in 40 minutes, and the teachers usually allow 10 minutes after assembly for you to get back to class. You estimate each speaker is about 6 minutes. If there are only speakers remaining in assembly, how many more speakers are there?



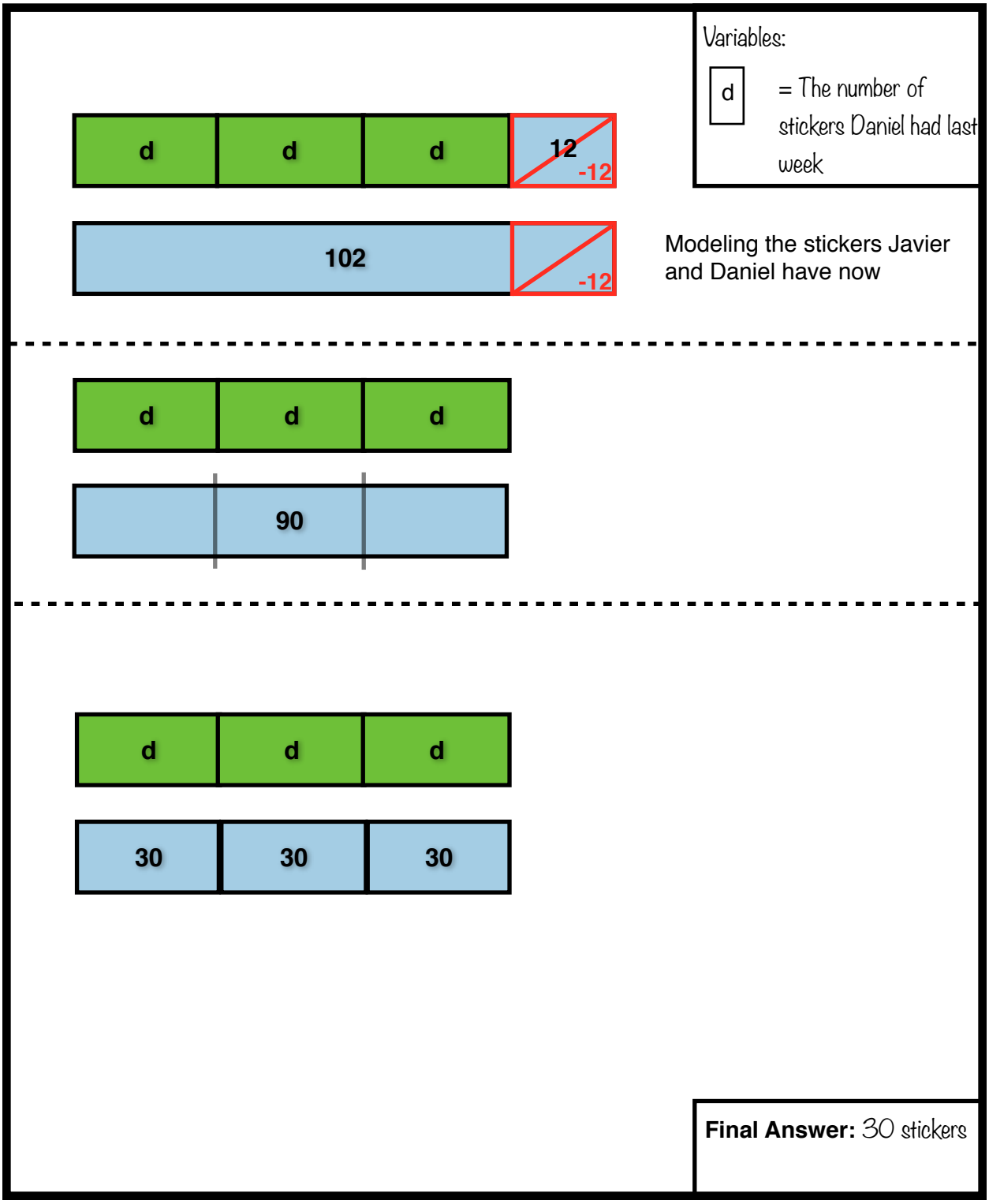
Problem: 4x + 7 = 19



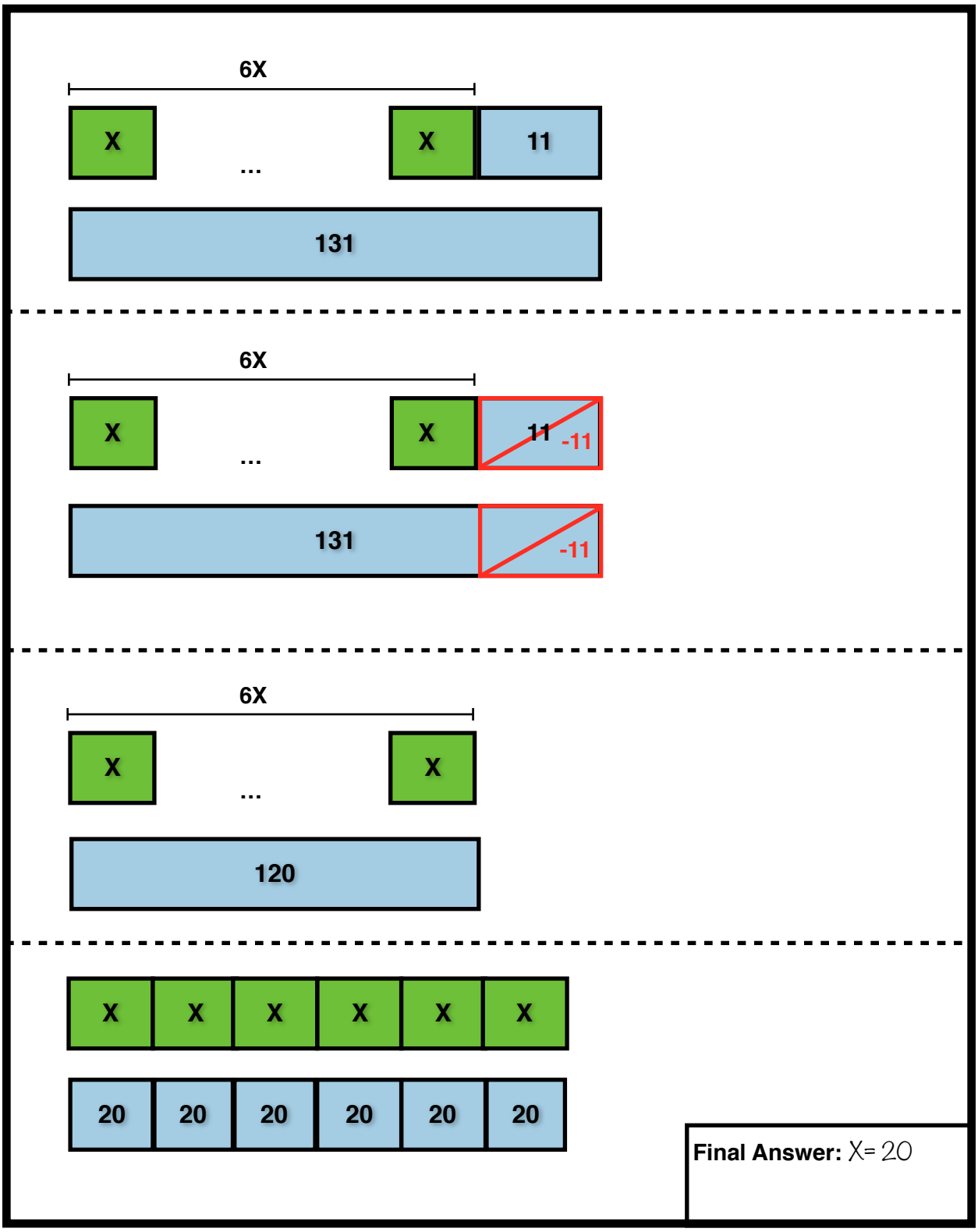
Problem: I am some number. Multiply me by 6, and add 4 and you get 10. What number am I?



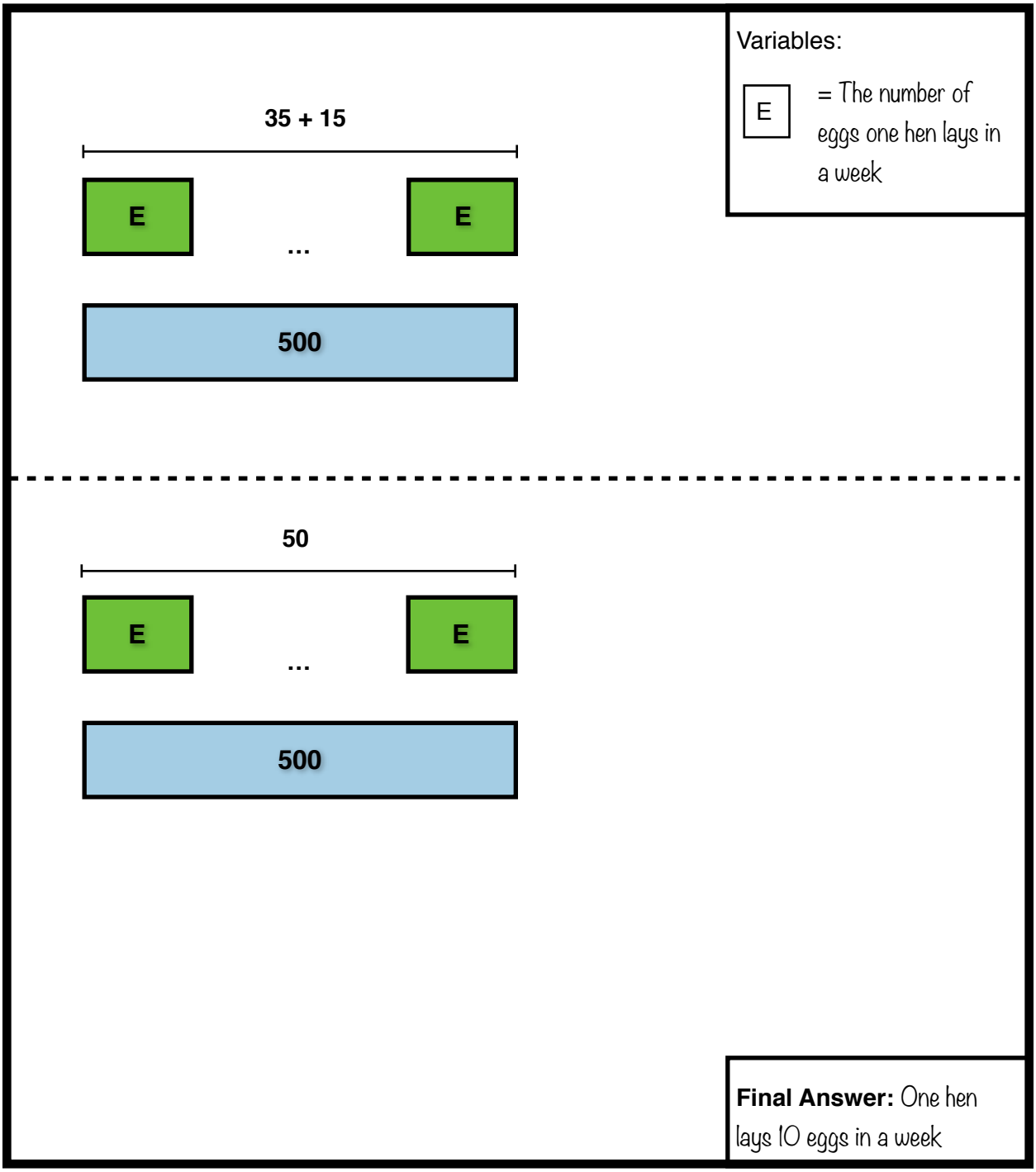
Problem: Last week Javier had twice as many stickers as Daniel. Then Daniel received 12 stickers for his birthday. Together they now have 90 stickers. How many stickers did Daniel have last week?



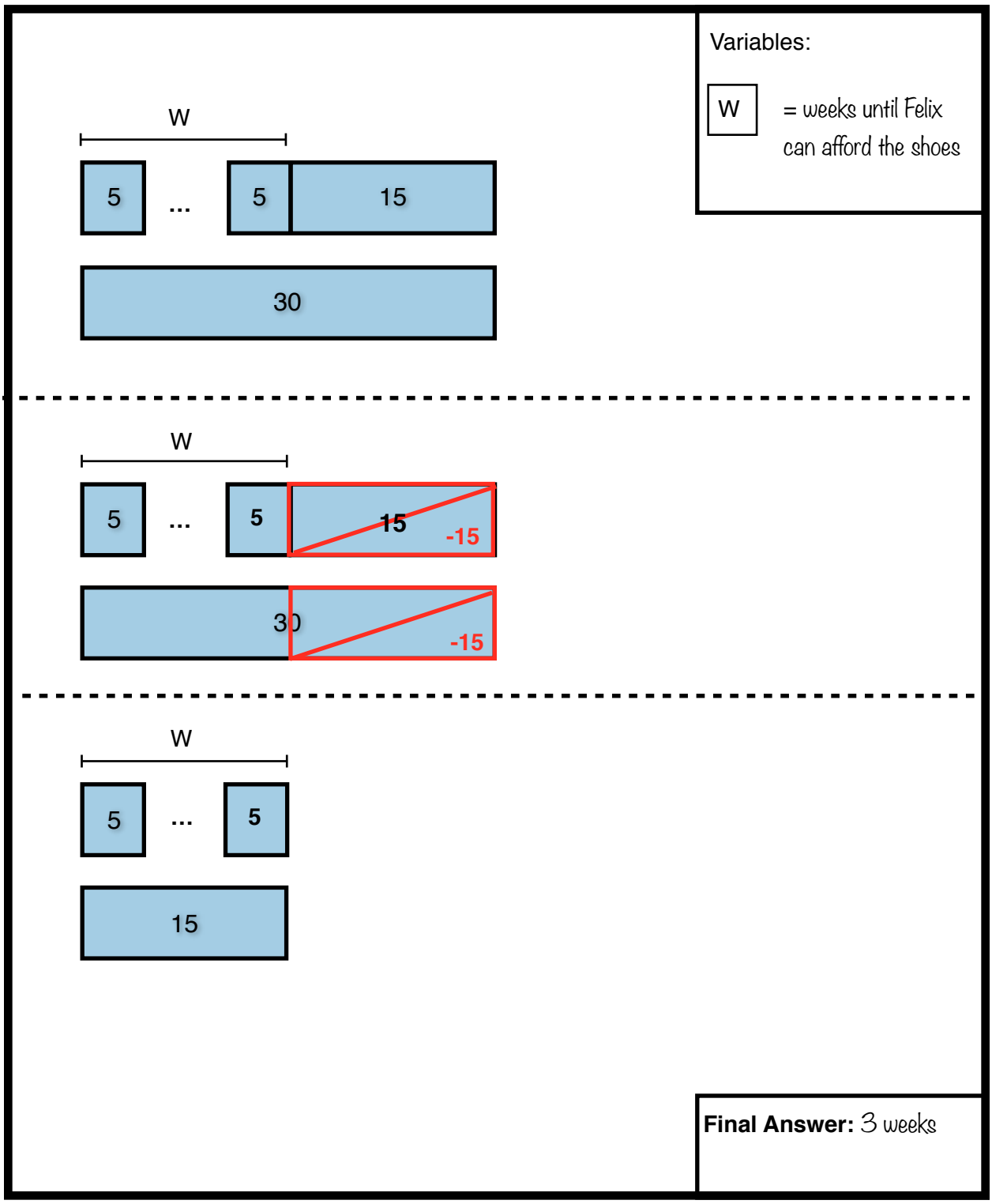
Problem: 131 = 6x + 11



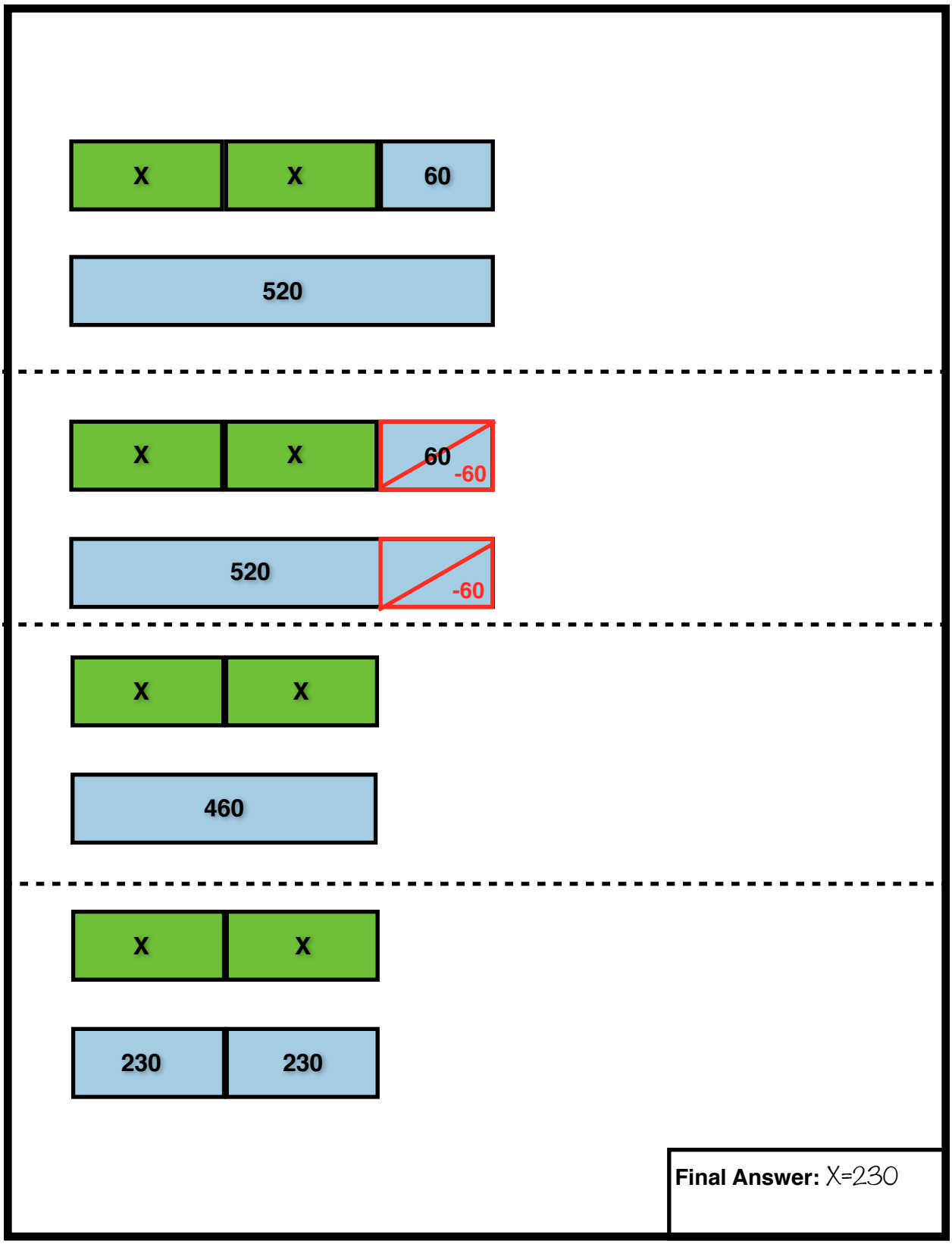
Problem: Sunny Hill Farms and Babbling Brooks Farms both raise hens. Sunny Hill Farms has 35 hens. Babbling Brooks Farms has 15 hens. If together the farms have 500 eggs at the end of the week, and each hen laid the same number of eggs, how many eggs did each hen lay last week?



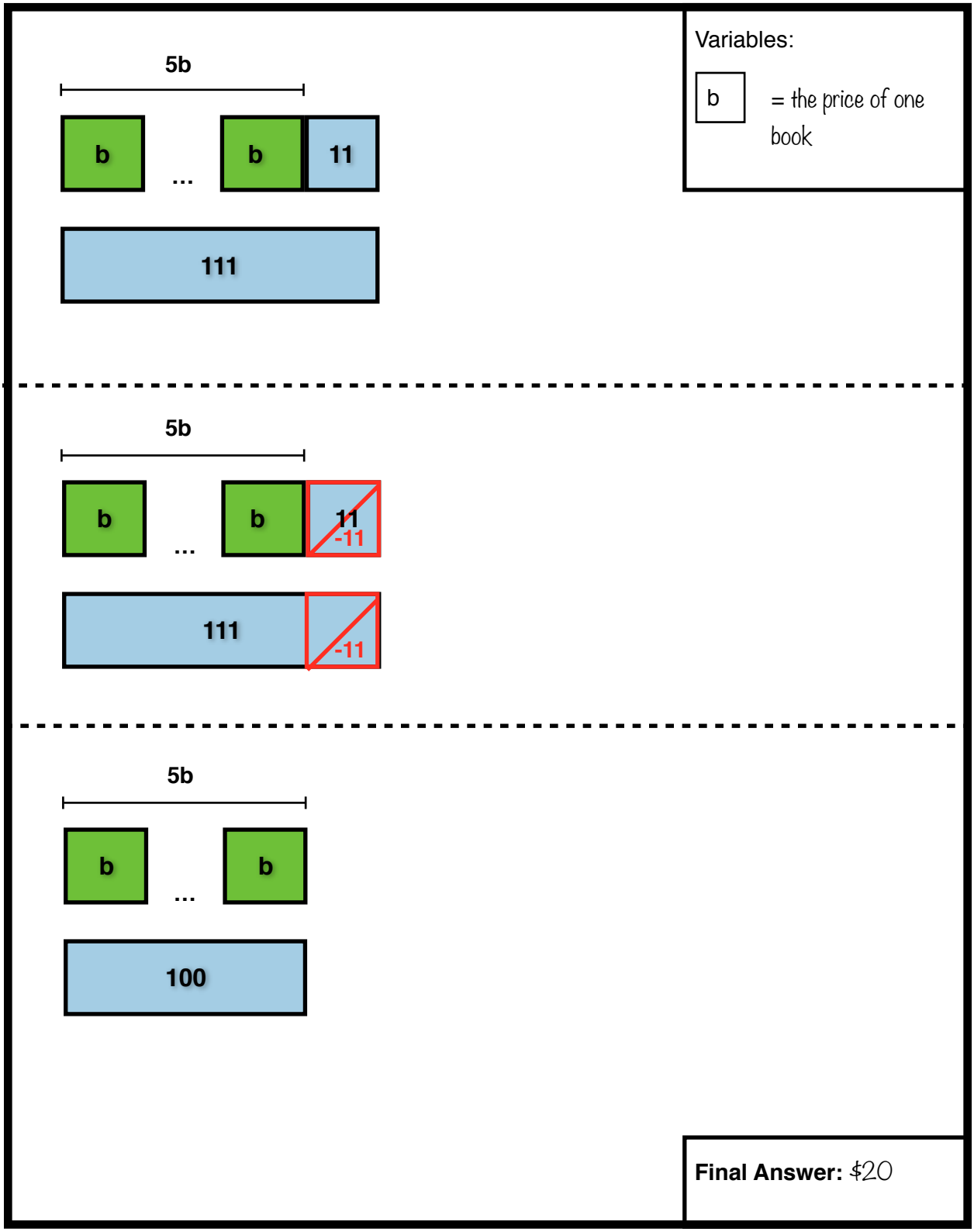
Problem: Felix wants to buy a new pair of shoes that cost $30. He has $15 now, and knows he can earn $5 a week helping his grandma with yard work. How many weeks will it take Felix to be able to afford the shoes?



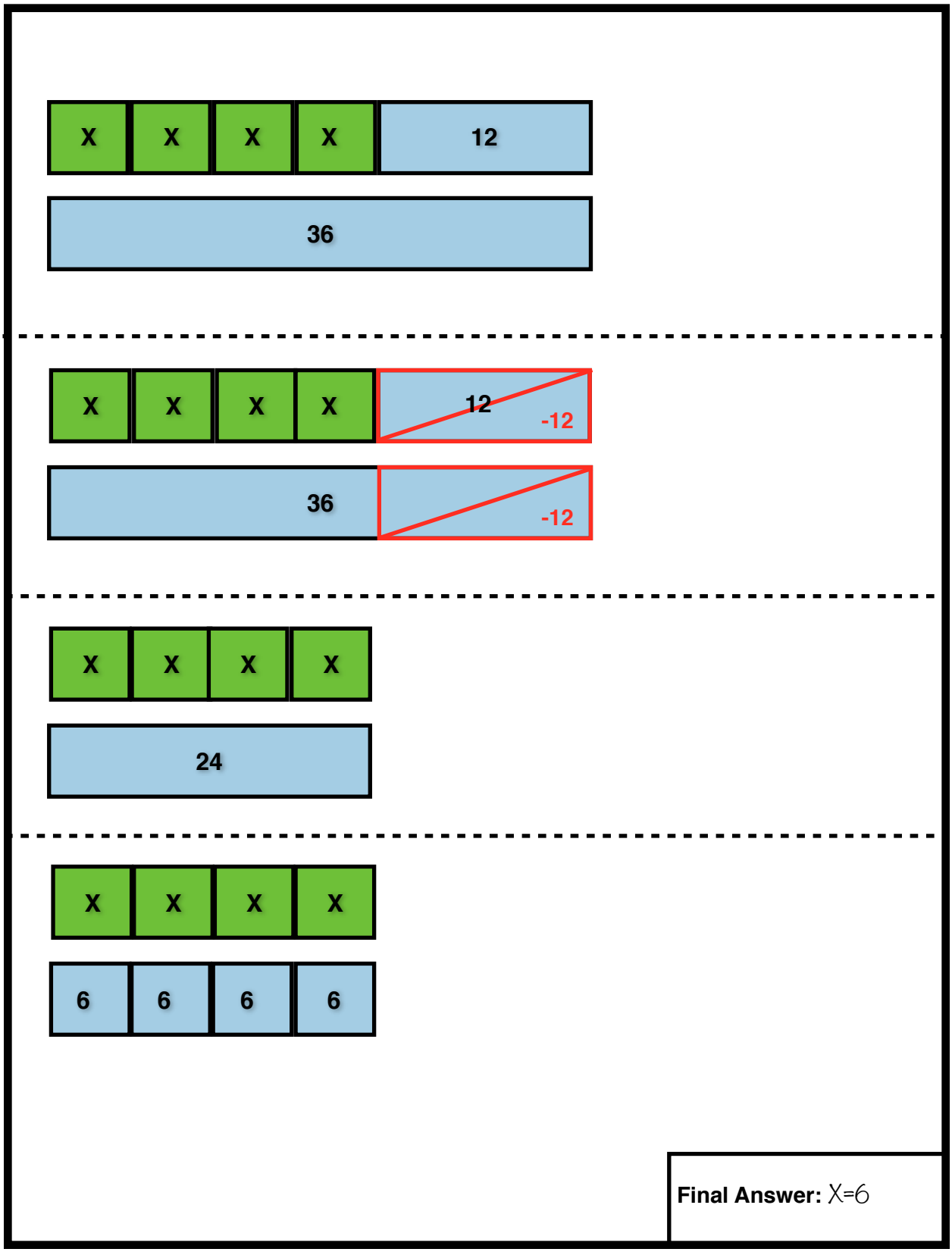
Problem: 520 = x + (x + 60)



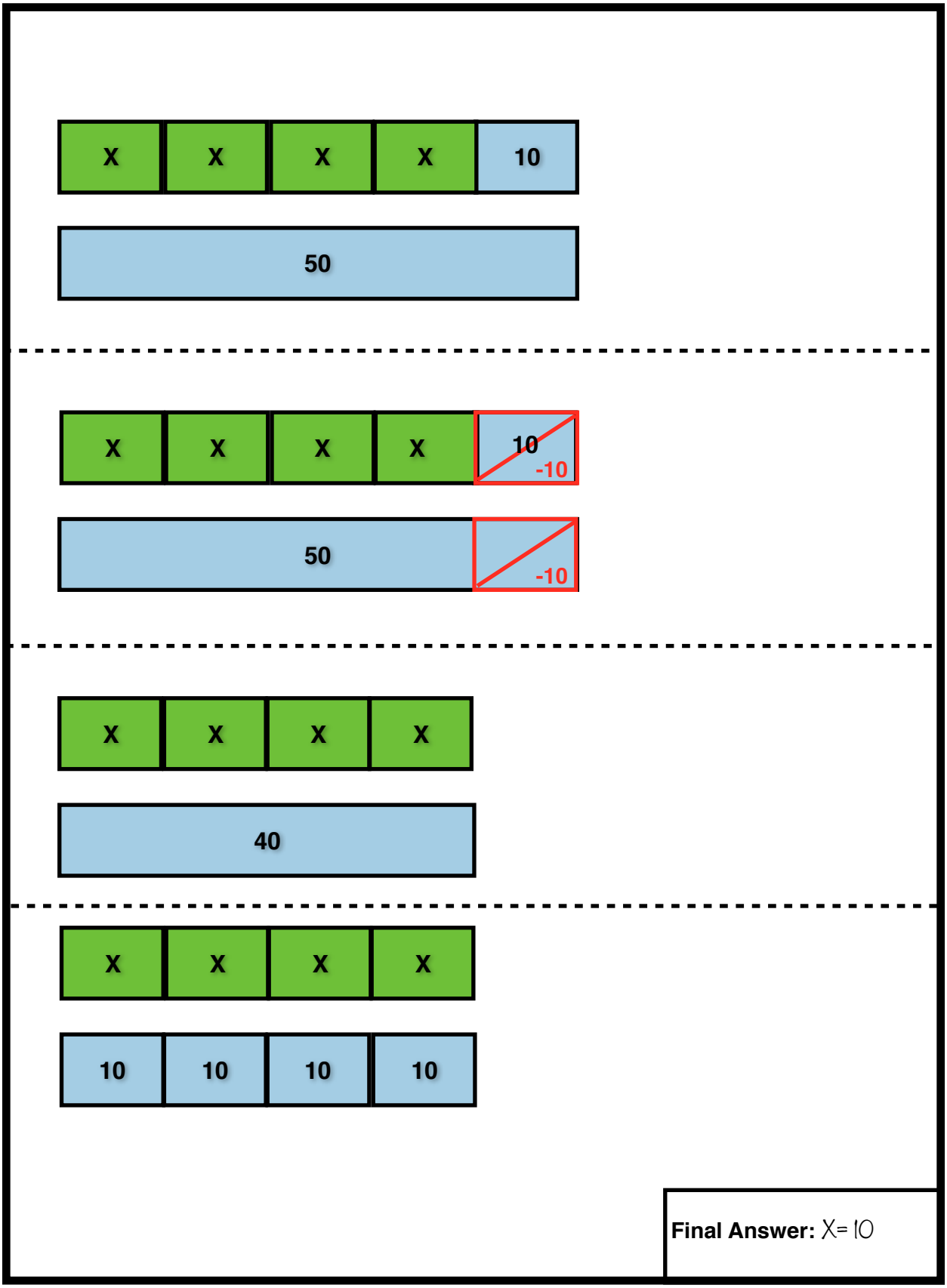
Problem: Jason had 111 dollars to spend. After buying 5 books he had 11 dollars left. Each book costs the same amount. How much did each book cost?



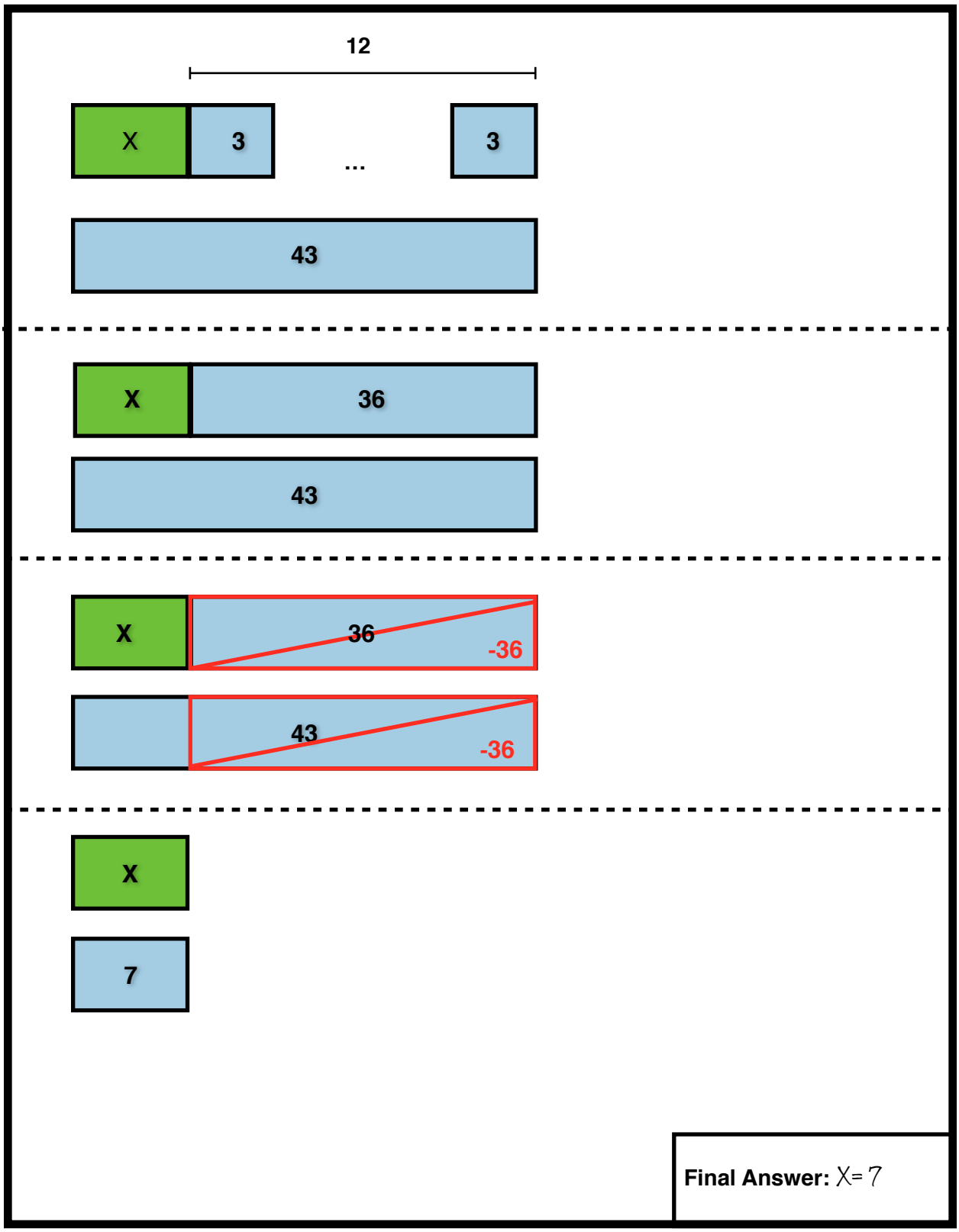
Problem: 36 = 12 + 4x



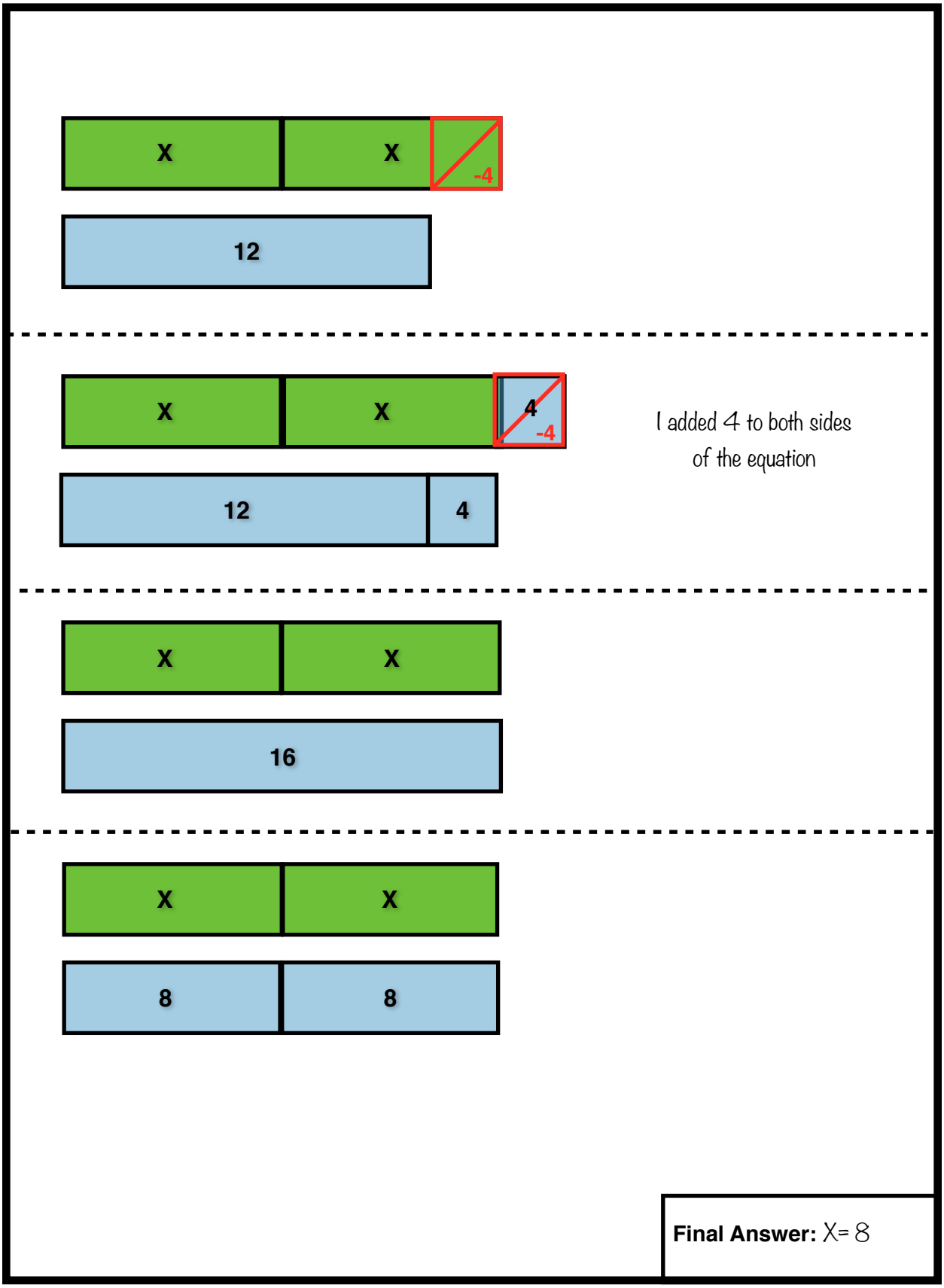
Problem: 50 = 10 + 4x



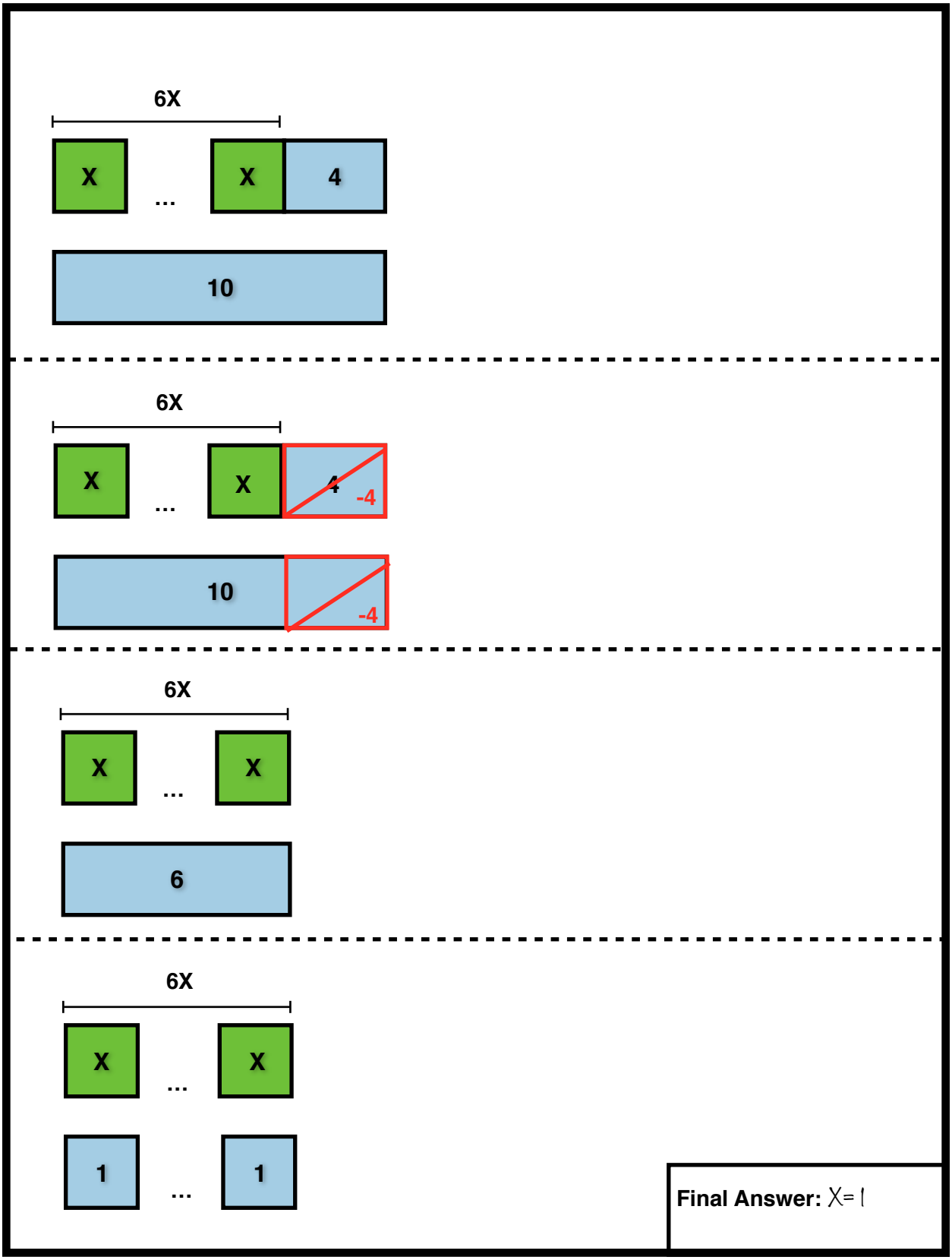
Problem: 12x3 + x = 43



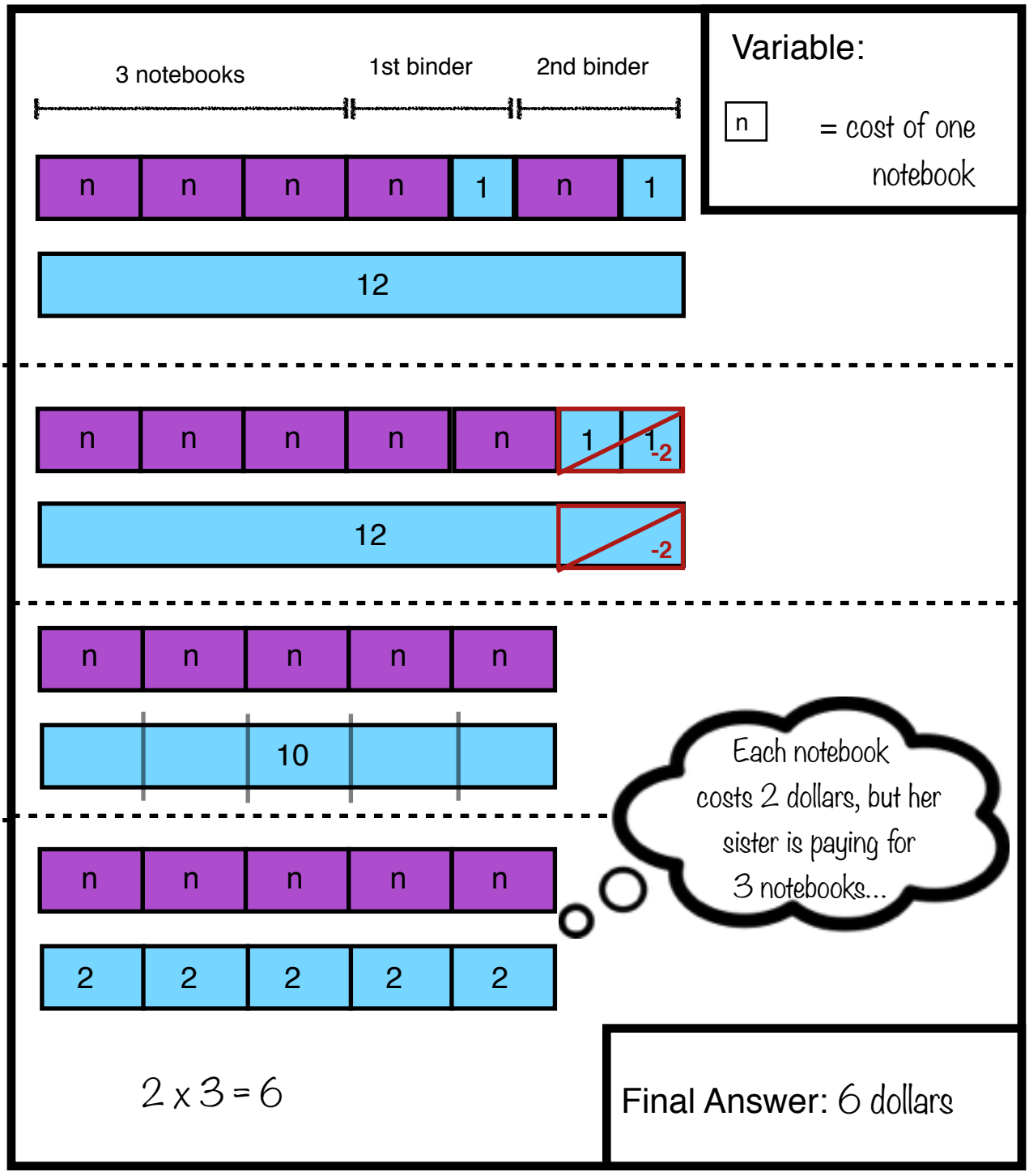
Problem: 12 = x + (x-4)



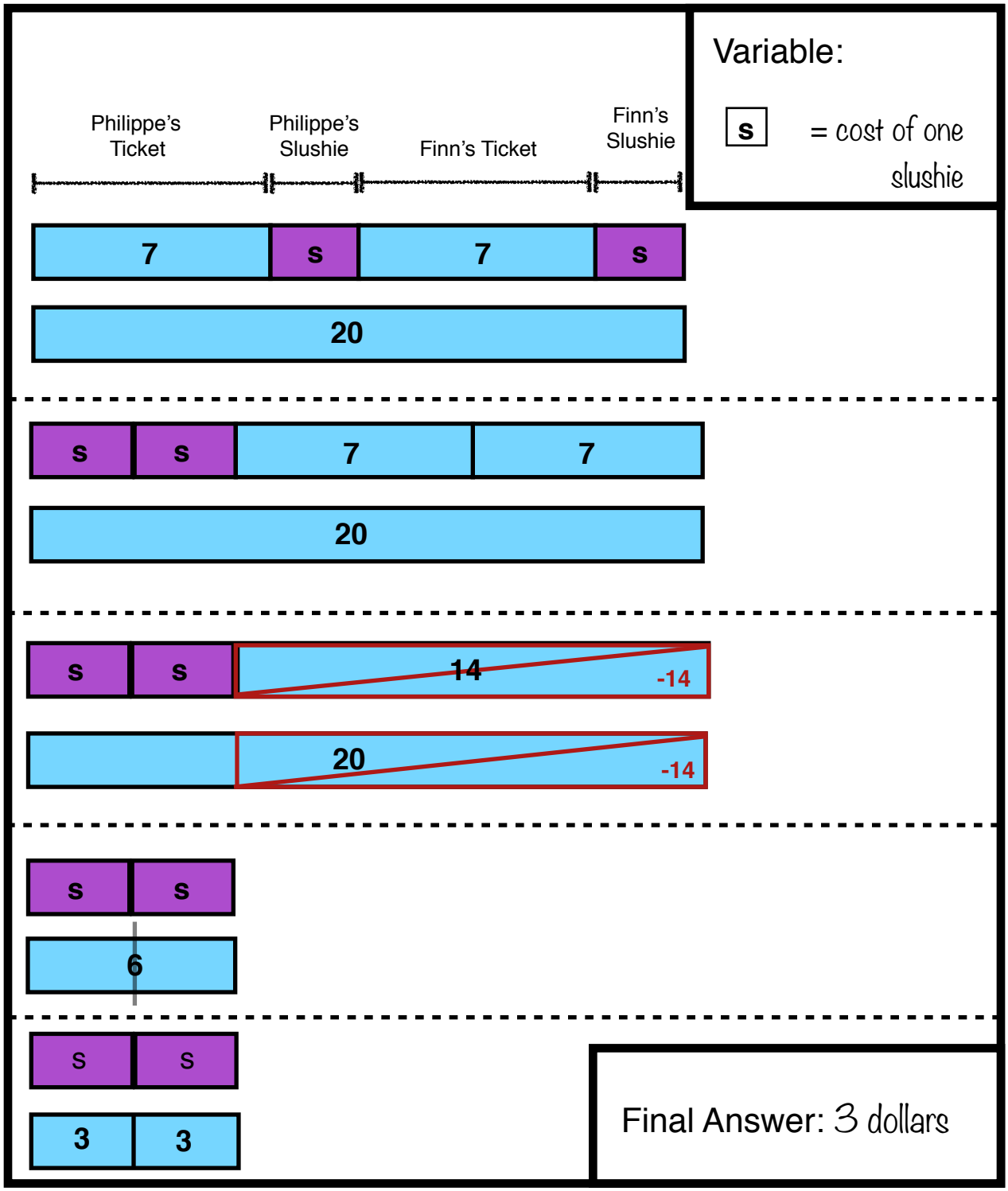
Problem: 6x + 4 = 10



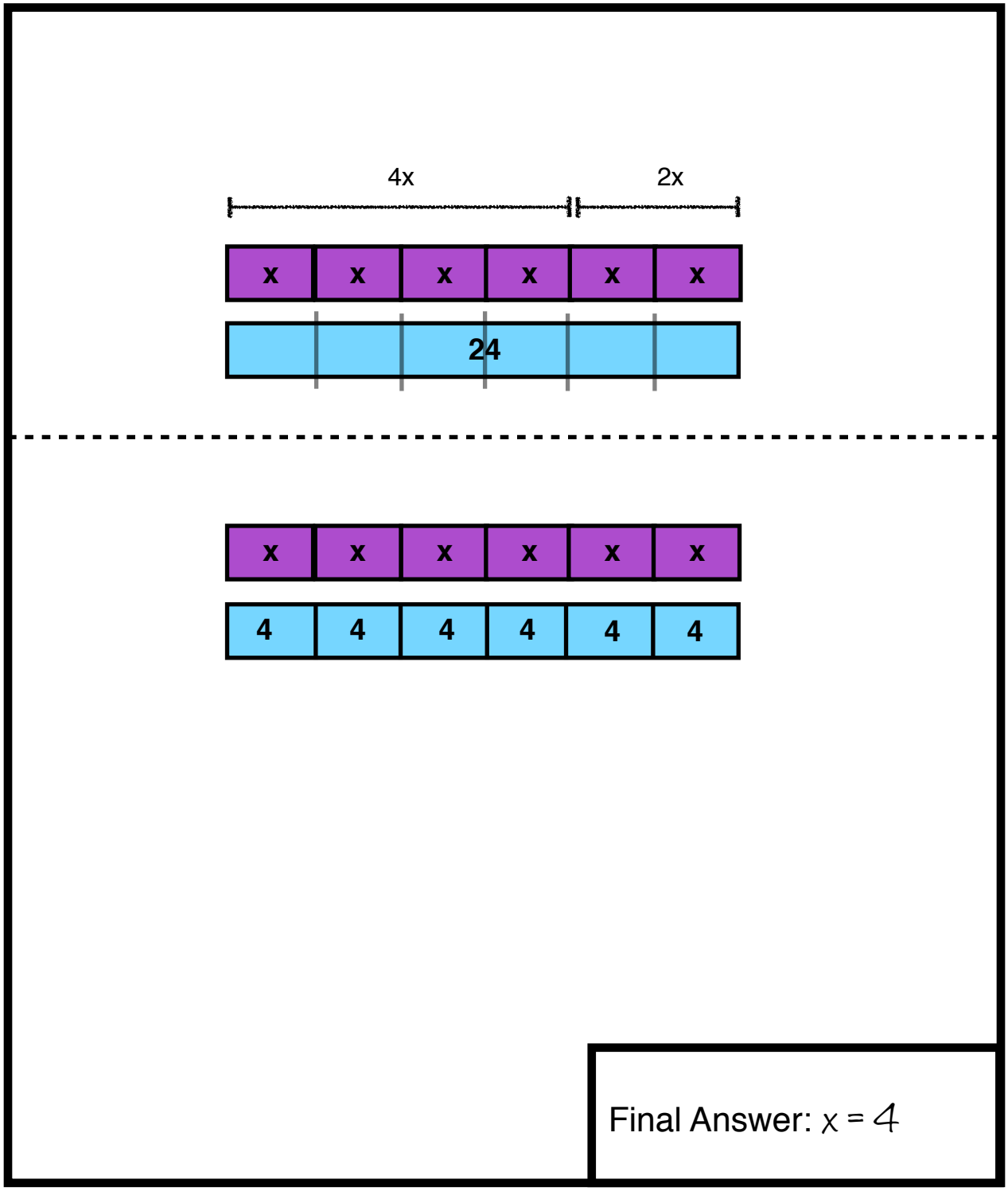
Problem: Alicia bought 3 notebooks and 2 binders, and her sister agreed to pay for the notebooks. Unfortunately, Alicia can't remember the cost of the notebooks! She does remember that each binder cost $1 more than each notebook and that she spent $12 total. How much does Alicia's sister need to pay Alicia?



Problem: Philippe and Finn go to see a movie. Each buys a ticket for $7 and a slushie, spending $18 together. How much does one slushie cost?

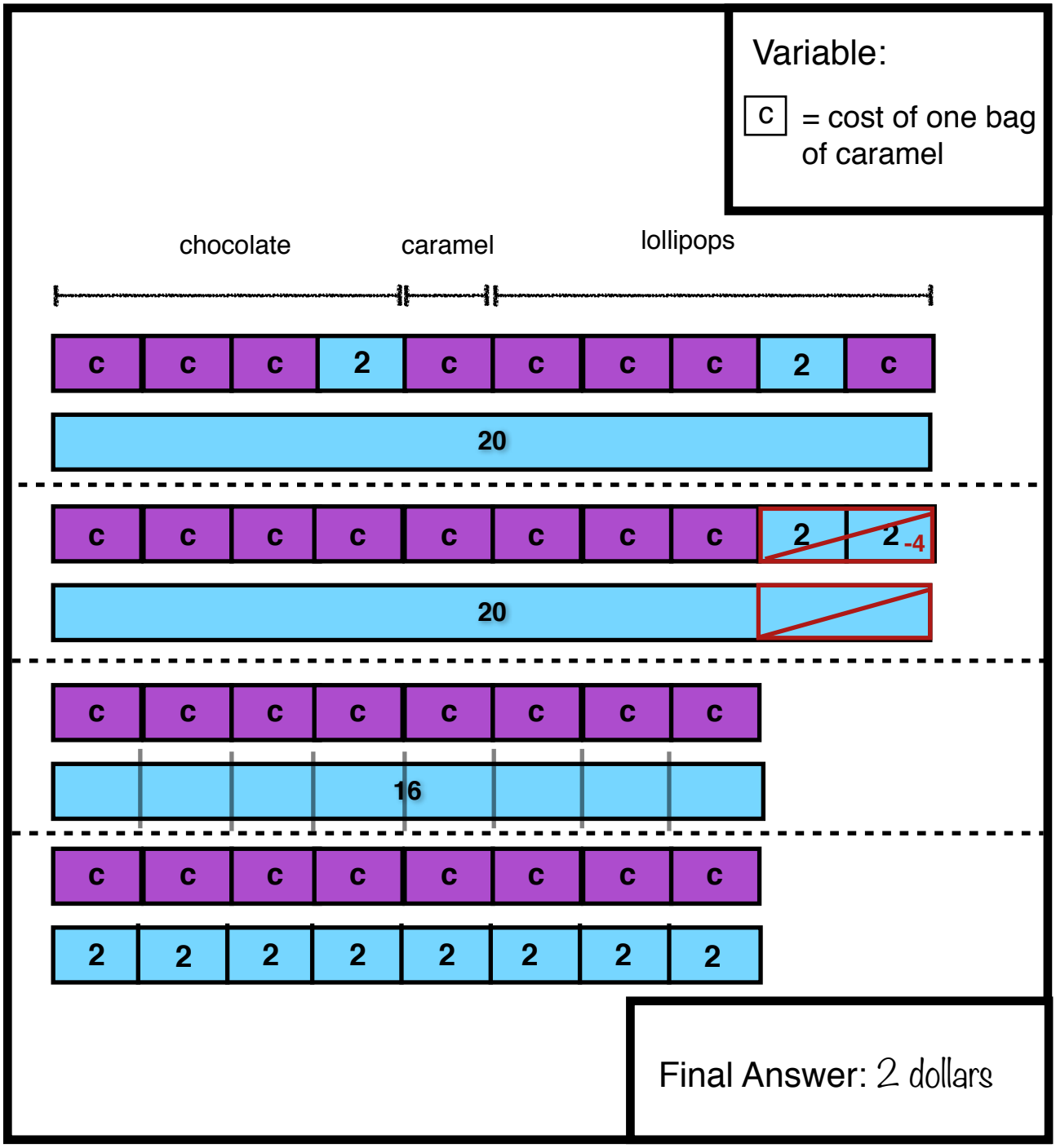


Problem: 4x + 2x = 24

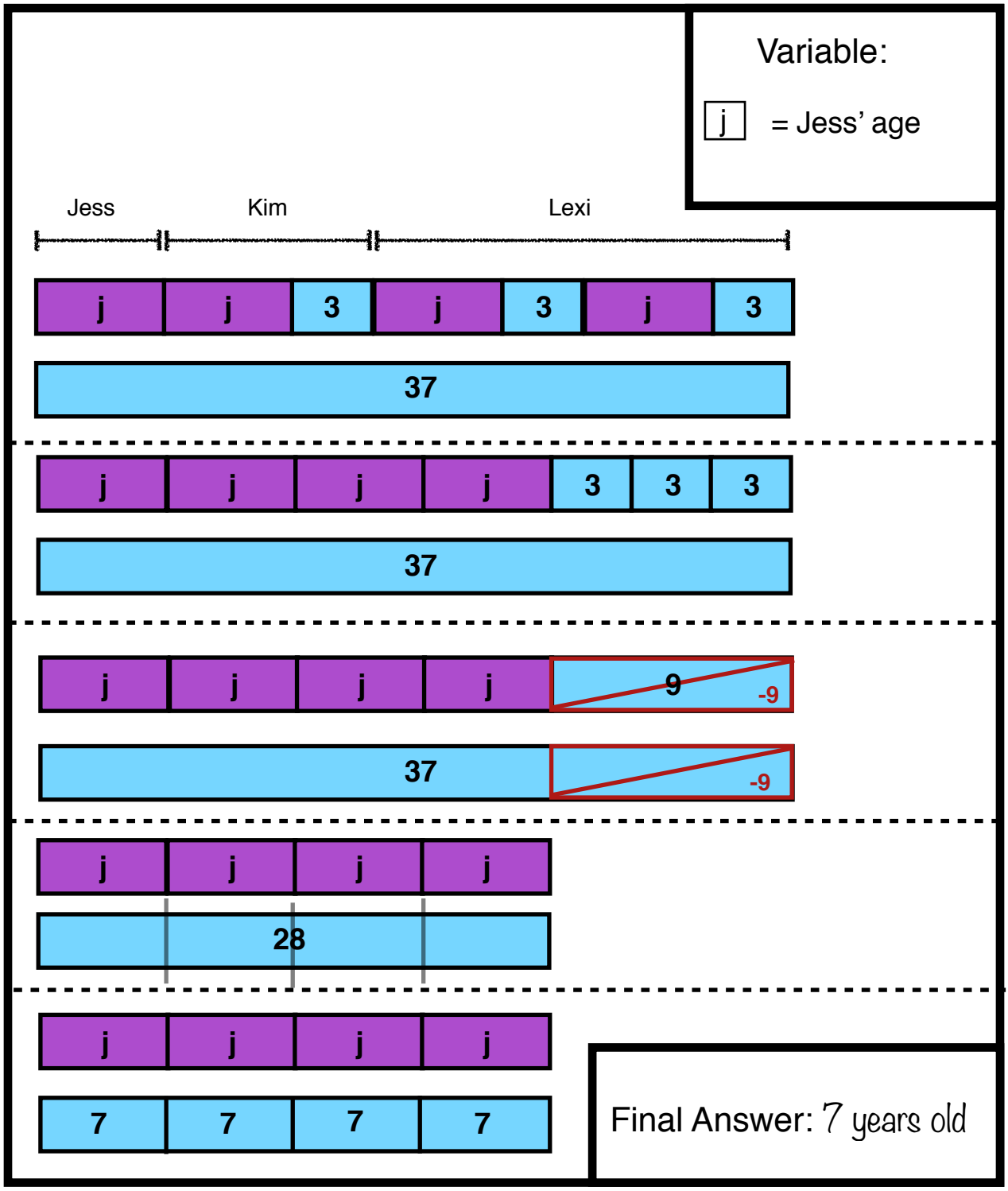


Final Answer: $x = 4$

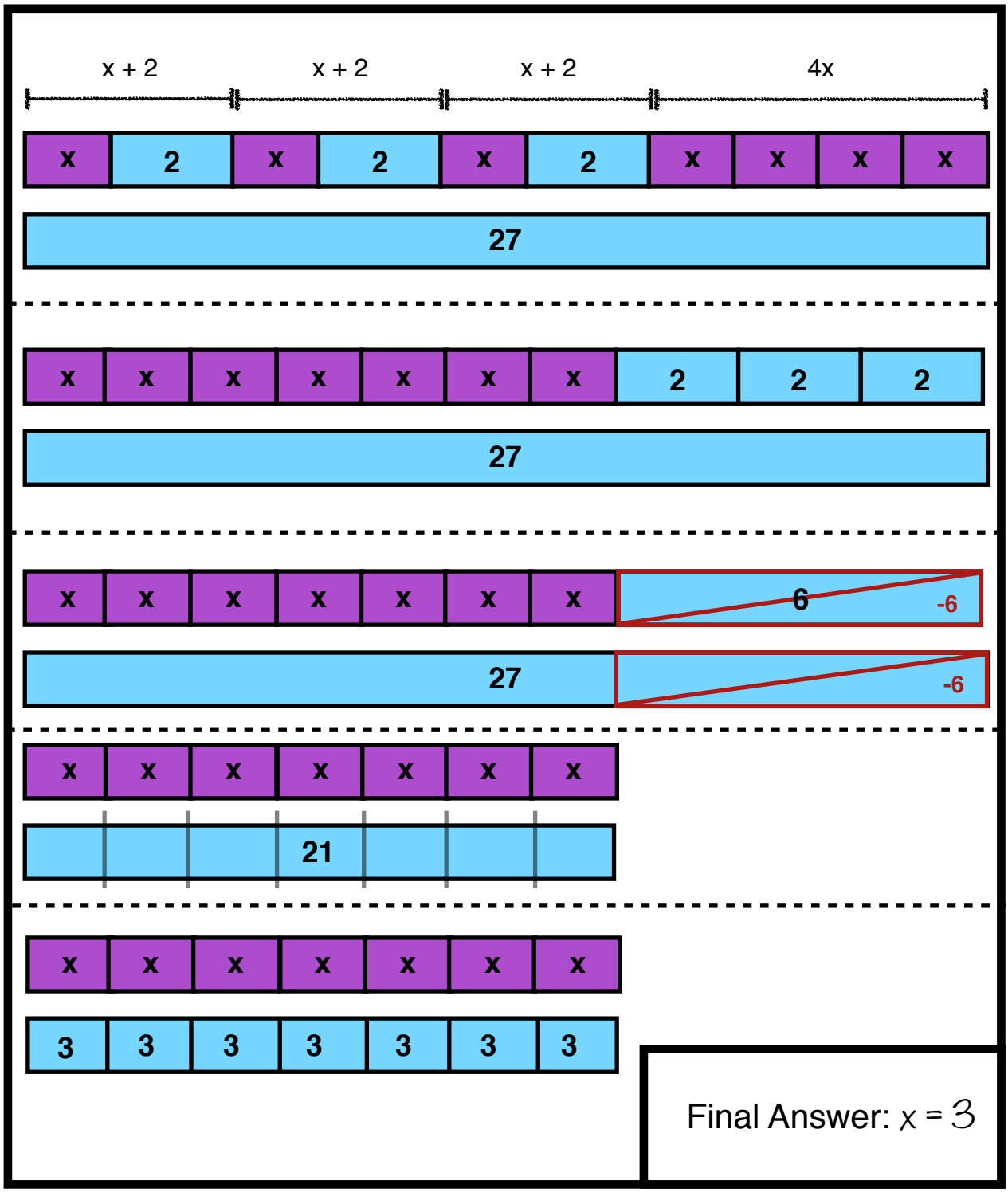
Problem: Jordan goes to the grocery store and buys one bag of chocolate, one bag of caramel, and one bag of lollipops. A bag of chocolate costs two dollars more than three times as much as a bag of caramel, and a bag of lollipops costs as much as buying a bag of caramel and a bag of chocolate. If Jordan spends 20 dollars for the three bags, how much does a bag of chocolate cost?



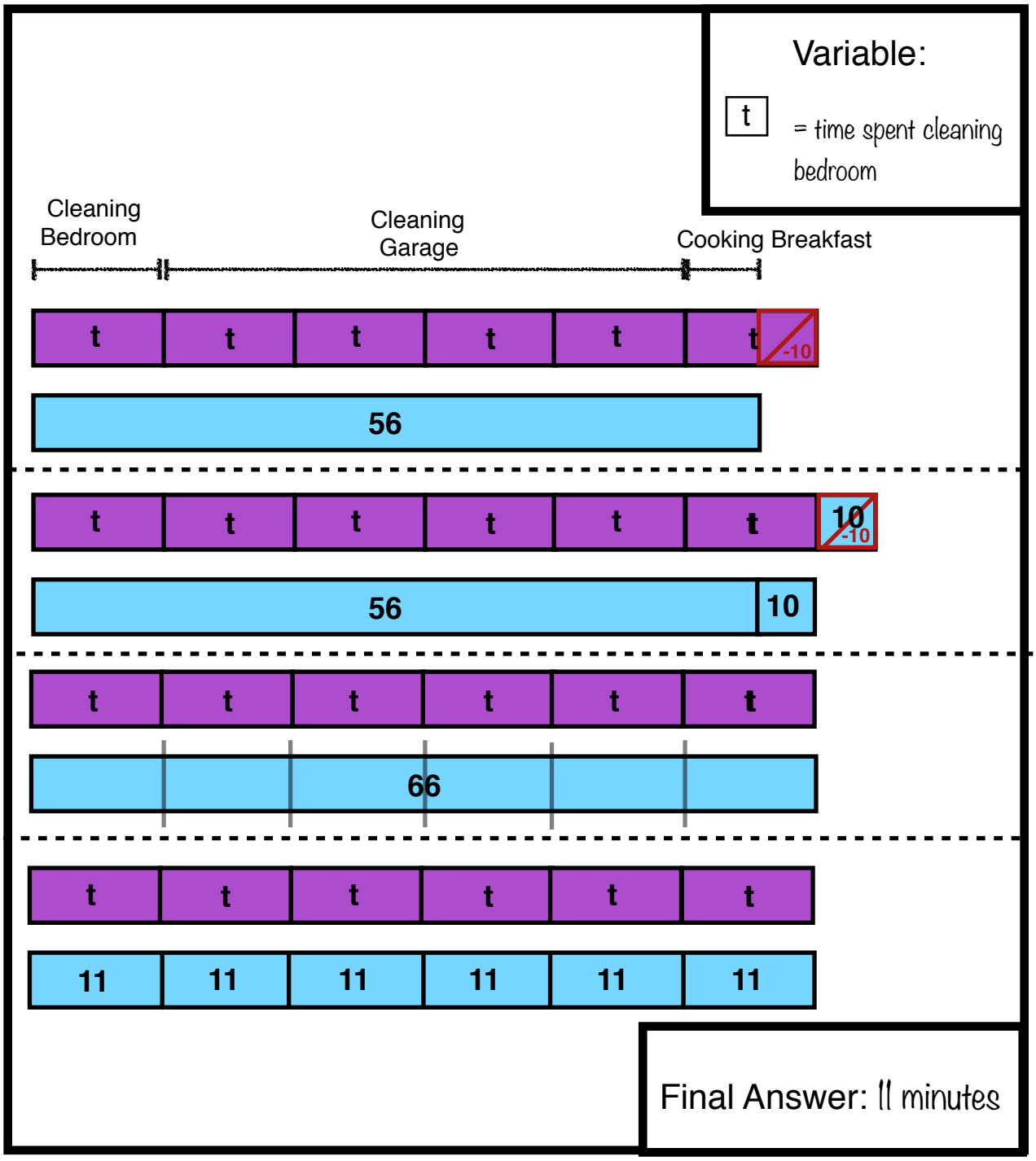
Problem: Kim is three years older than her sister Jess and half as old as her cousin Lexi. If they add their ages together, they get 37. How old is Jess?



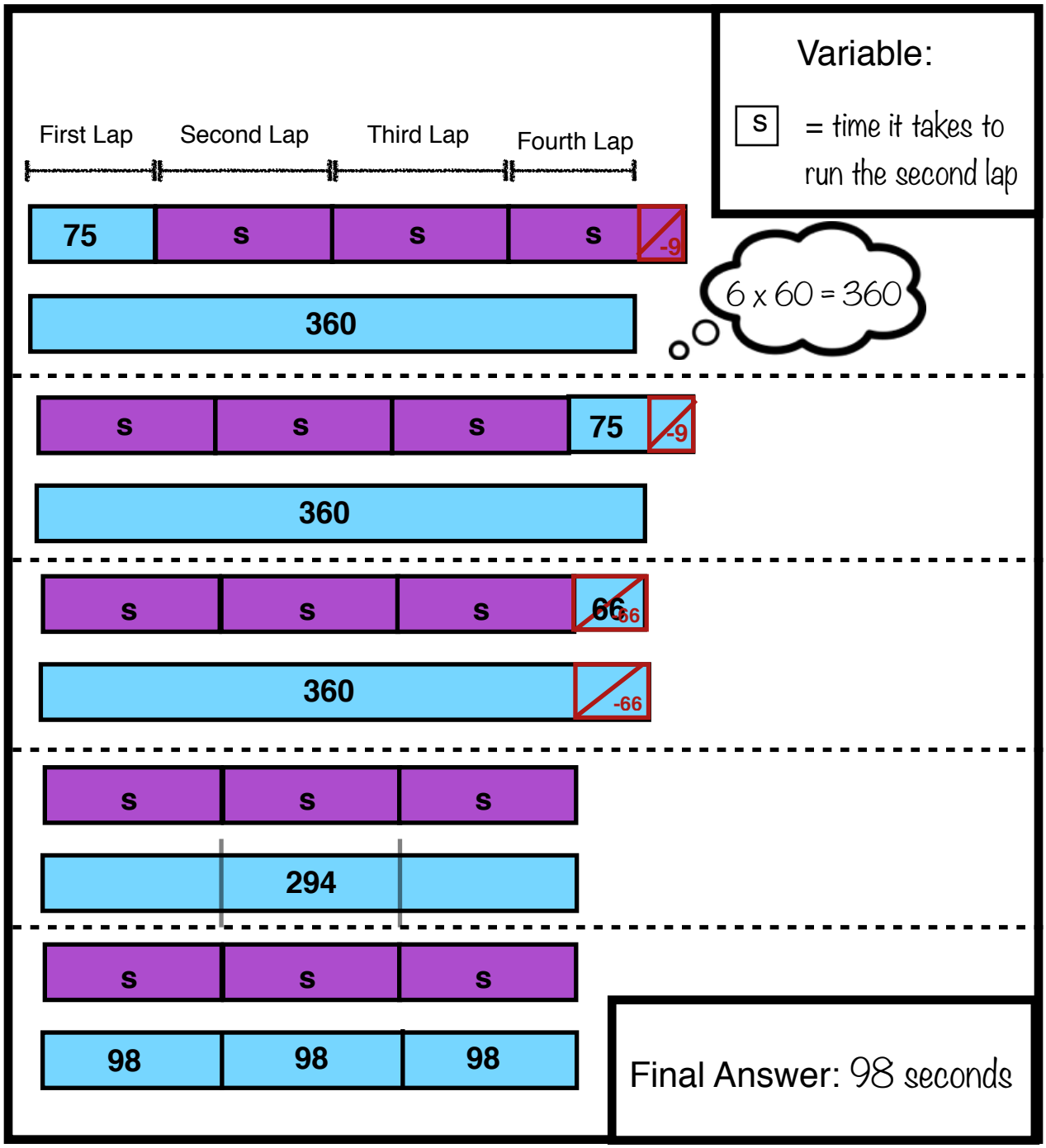
Problem: 3(x+2) + 4x = 27



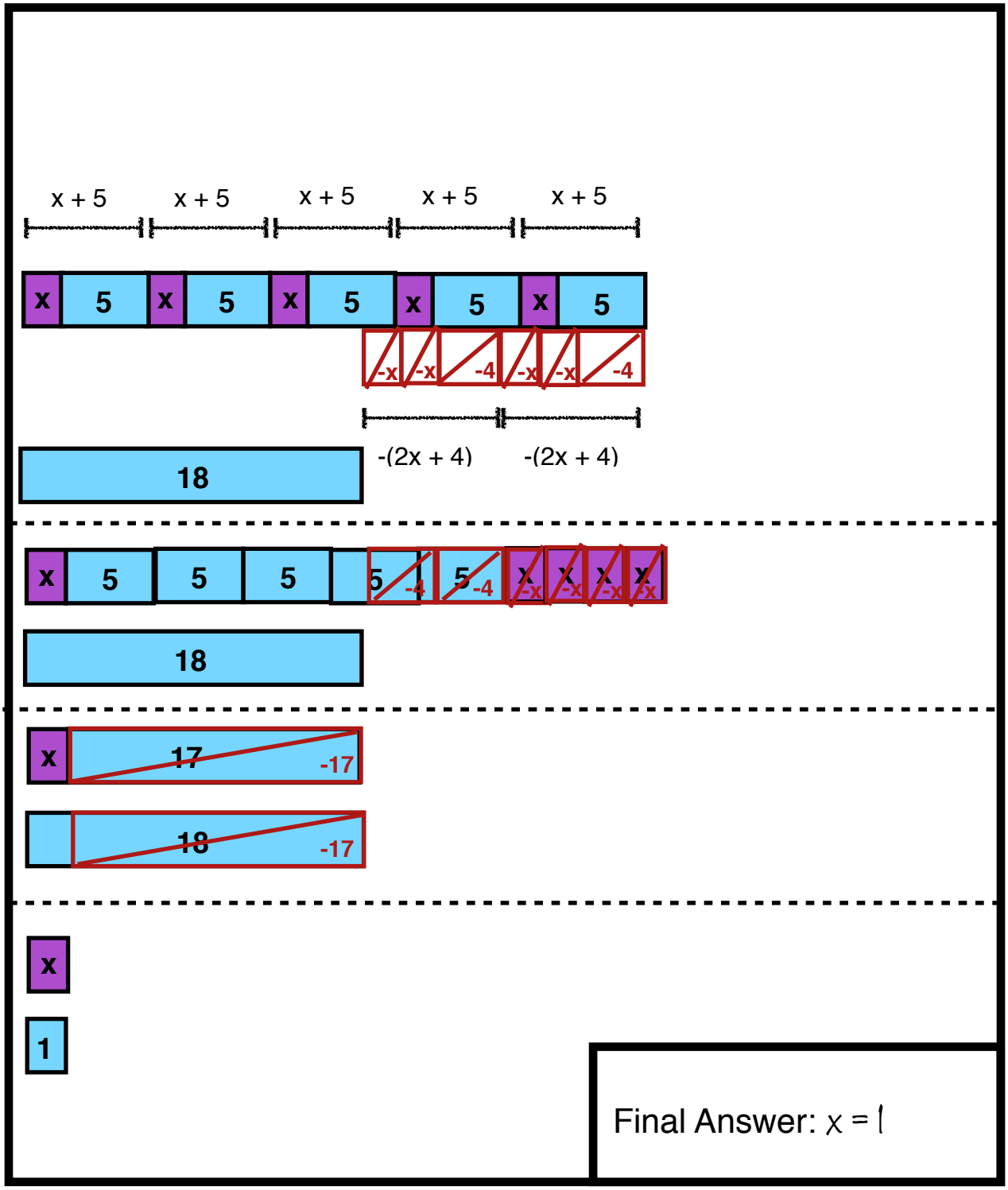
Problem: Jeremiah had a busy Saturday morning! Starting at 9:00 am, he cleaned his bedroom for a while. Then he helped his mom clean the garage for four times as long as he had spent cleaning his room. Finally he cooked breakfast with his sister for ten minutes fewer than the time he had spent cleaning his room, until 9:56am. How many minutes did Jeremiah spend cleaning his room in the morning?



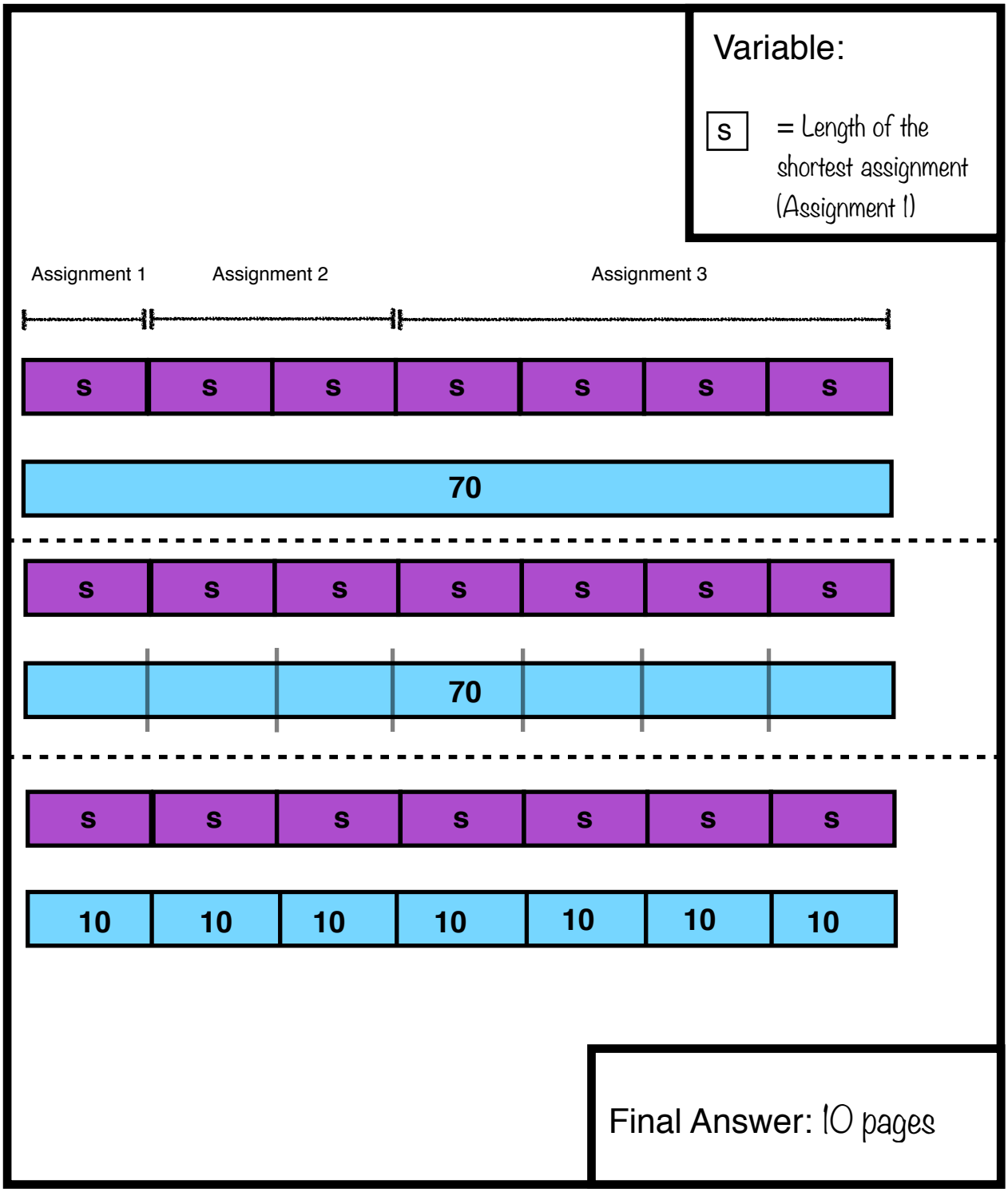
Problem: Izumi is running the mile (4 laps) at a track meet. She knows that she can run her first lap in 75 seconds. Izumi also knows that her second and third laps are the same speed, while her final lap is normally 9 seconds faster than her third lap. If she wants to finish in 6 minutes, how fast should her second lap be? (Note: there are 60 seconds in 1 minute)



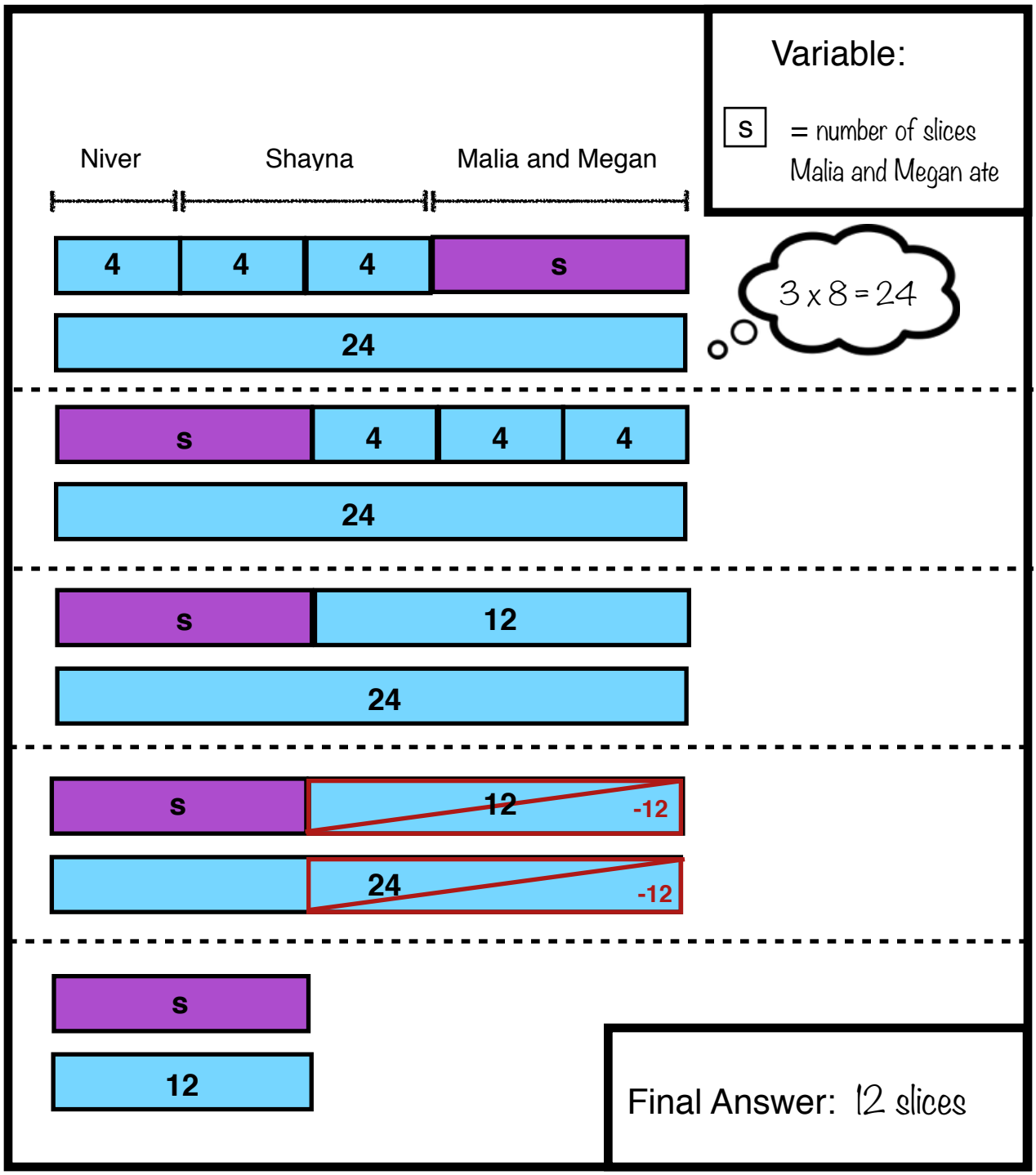
Problem: 5(x+5) - 2(2x+4) = 18



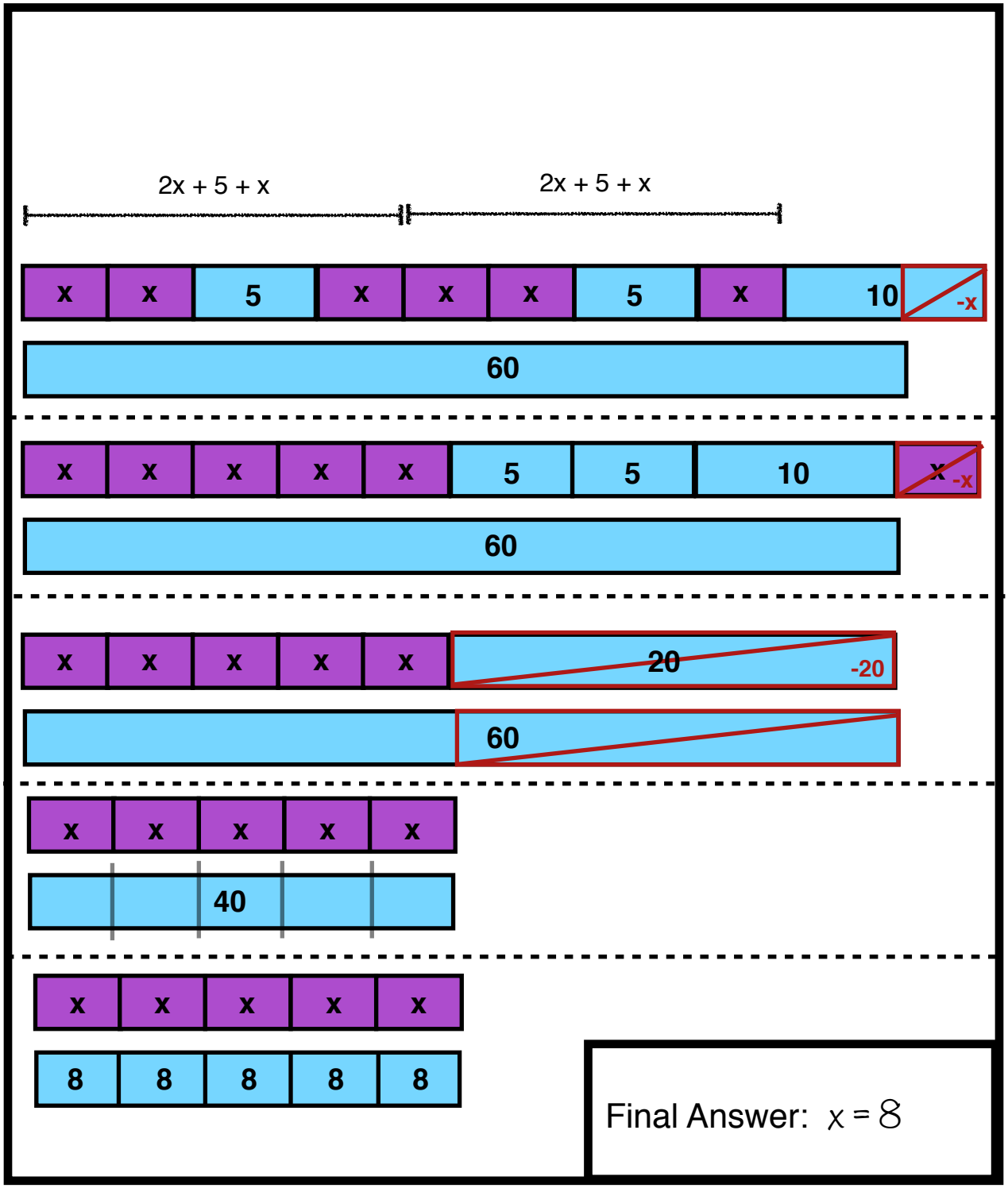
Problem: Jamal has three reading assignments to complete. In total he has to read 70 pages. Assignment 2 is twice as long as assignment 1, and assignment 3 is four times long as assignment 1. How many pages is his shortest assignment?



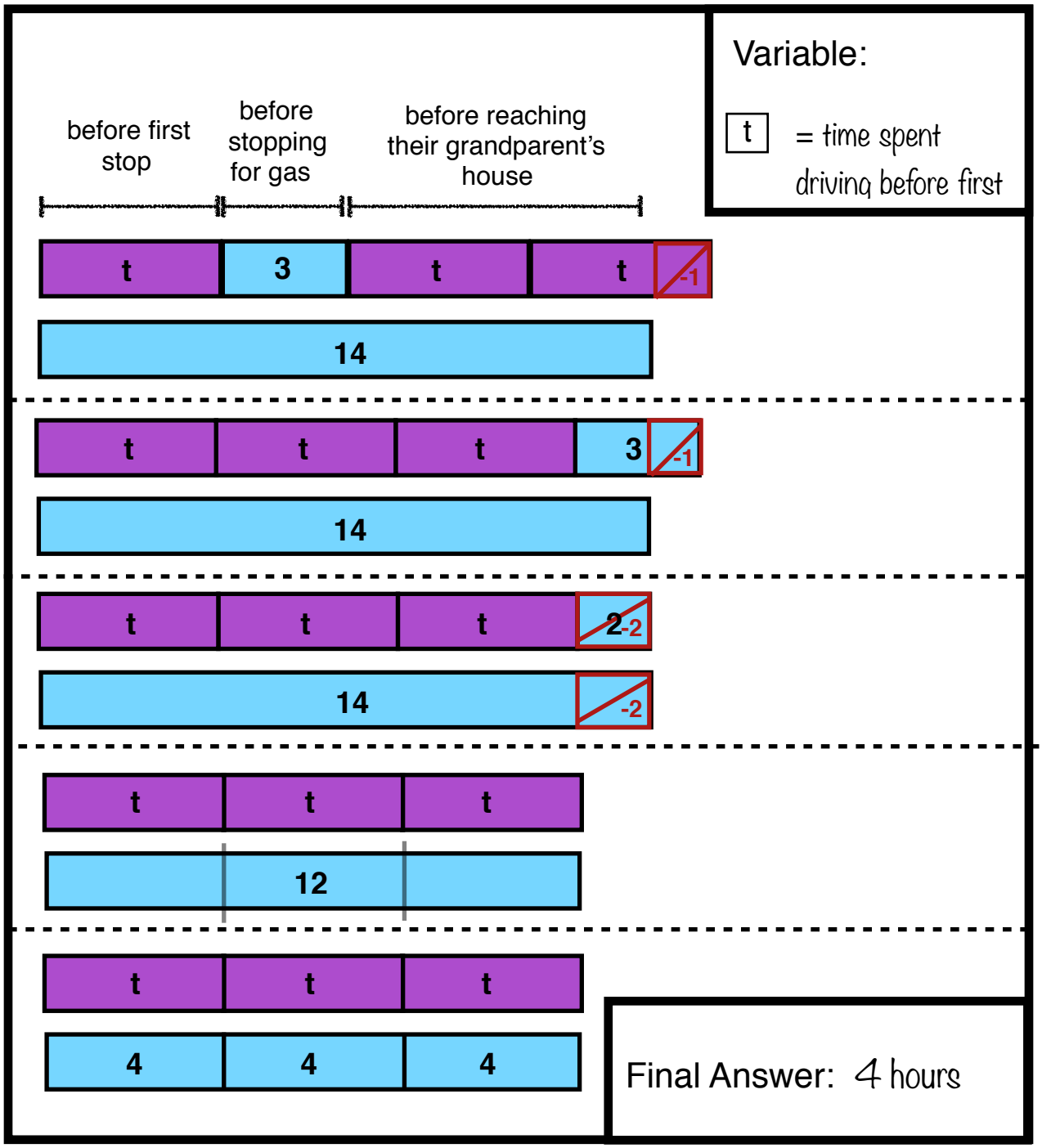
Problem: Malia and Megan ordered 3 pizzas and each pizza had 8 slices. Their friend Niver ate 4 slices of pizza, their friend Shayna ate twice as many pieces as Niver. Malia and Megan ate all of the remaining slices. How many slices did Malia and Megan eat?



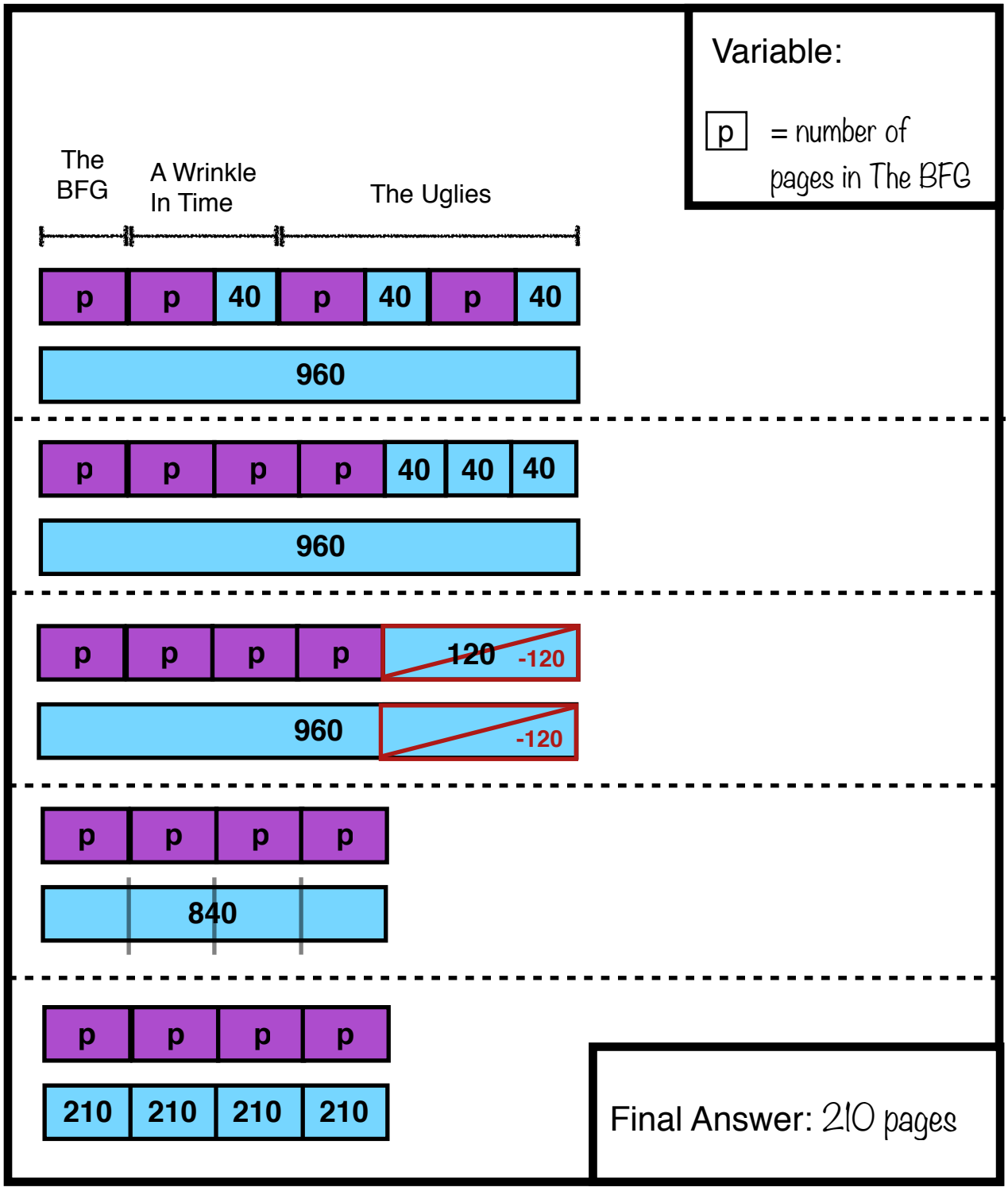
Problem: 2(2x + 5 + x) + 10 - x = 60



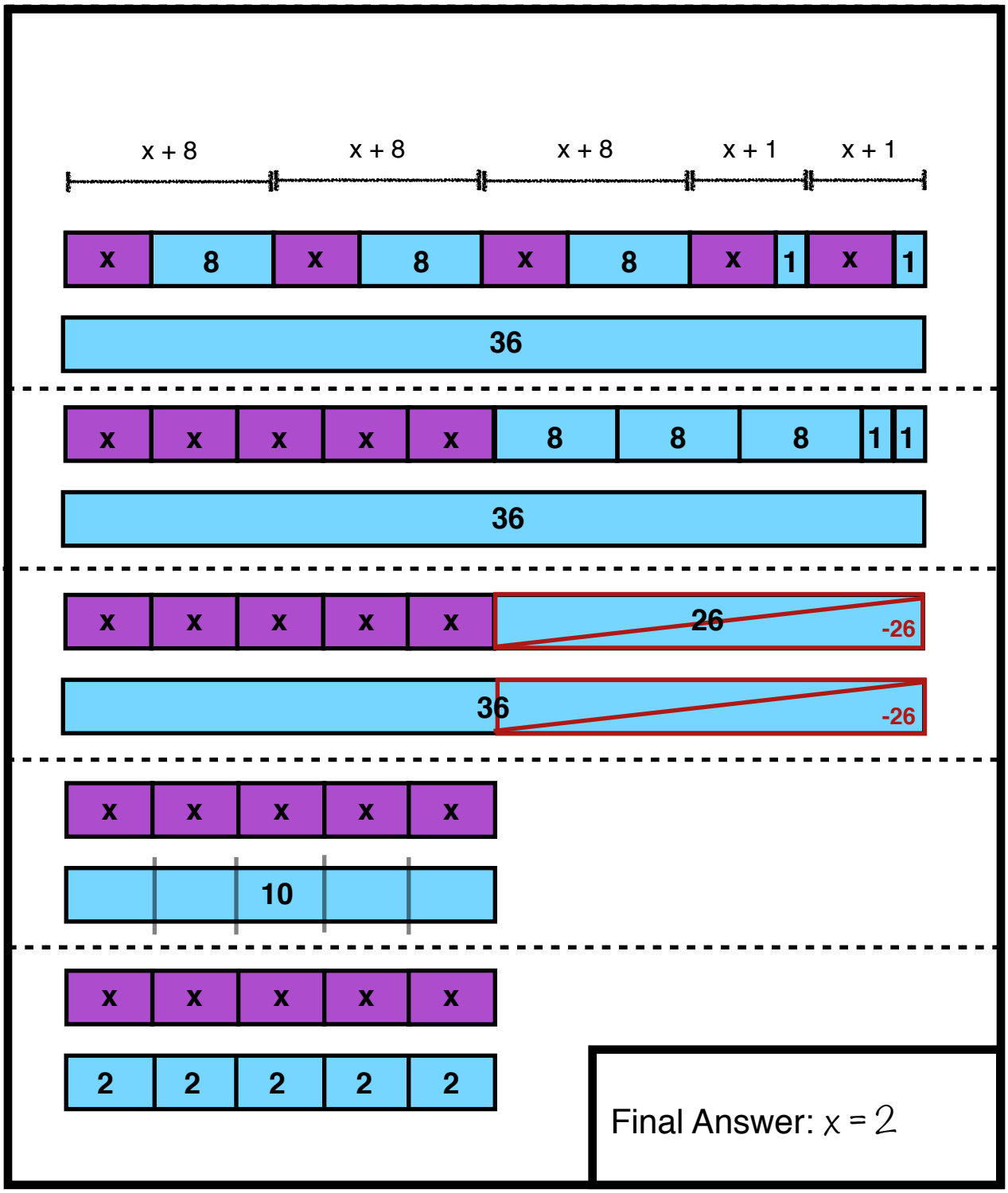
Problem: Shailee and Sofia are on a roadtrip to see their grandparents. They drive for a while before stopping for lunch. After that, they drive again for 3 hours before getting gas. Before reaching their grandparents house, they drive 1 hour less than twice as long as they drove before the first stop. In total, they drove 14 hours on their trip. How long did they drive before the first stop, for lunch?



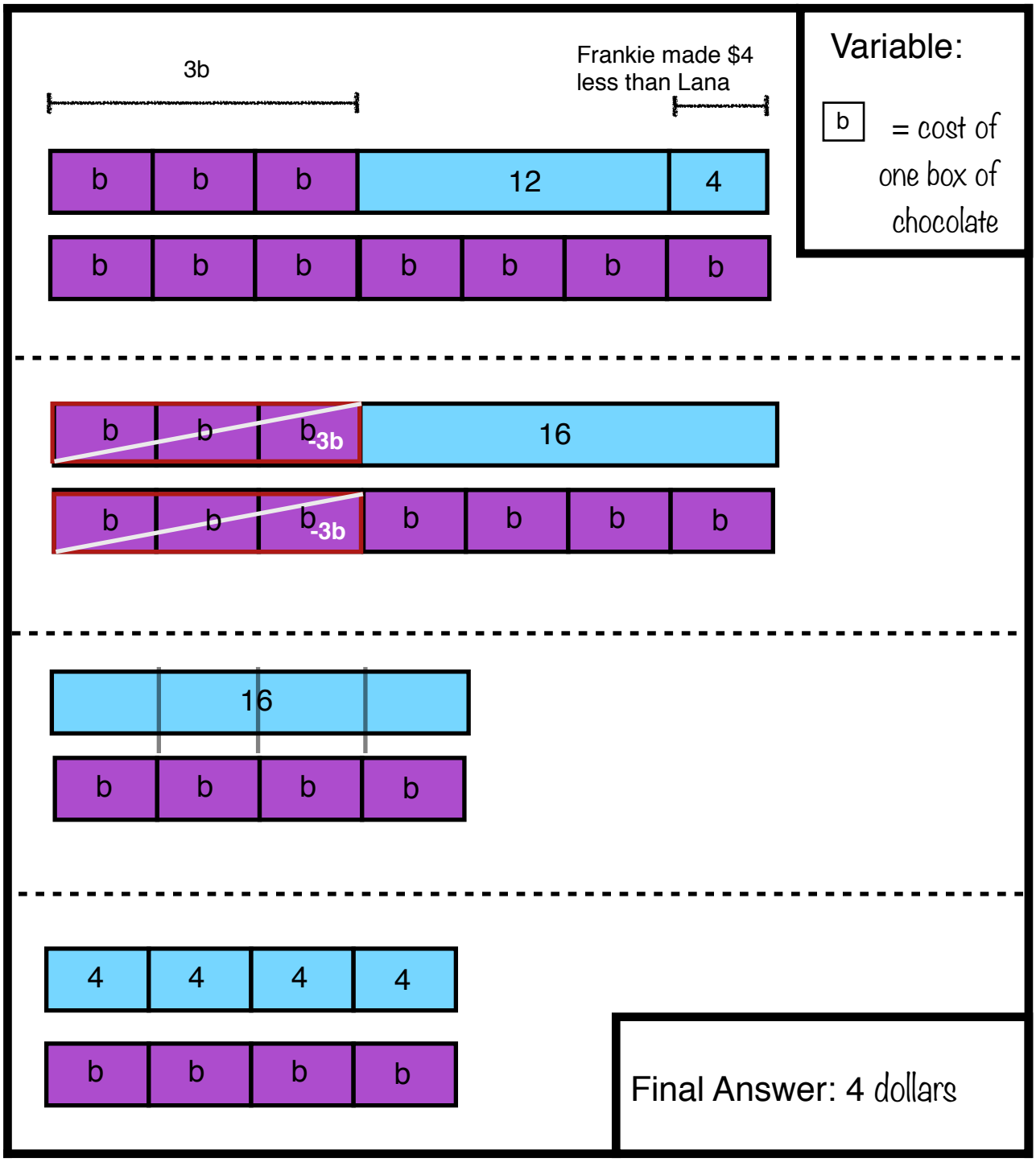
Problem: Marcos picked up three books from the library. *The Uglies* is twice as long as *A Wrinkle In Time*, and *A Wrinkle In Time* is forty pages longer than *The BFG*. Altogether, the three books have 960 pages. How many pages long is *The BFG*?



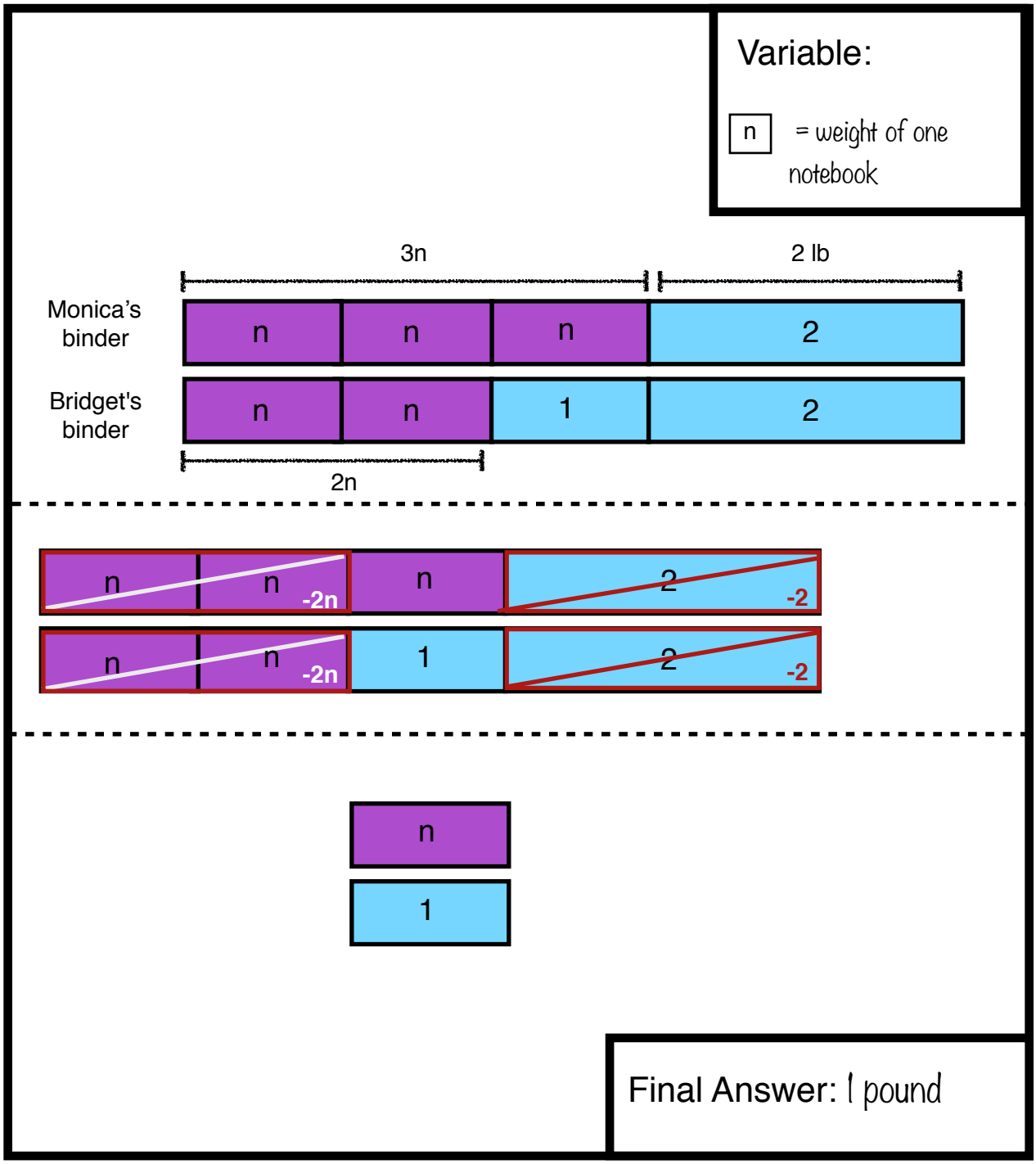
Problem: 3(x + 8) + 2(x + 1) = 36



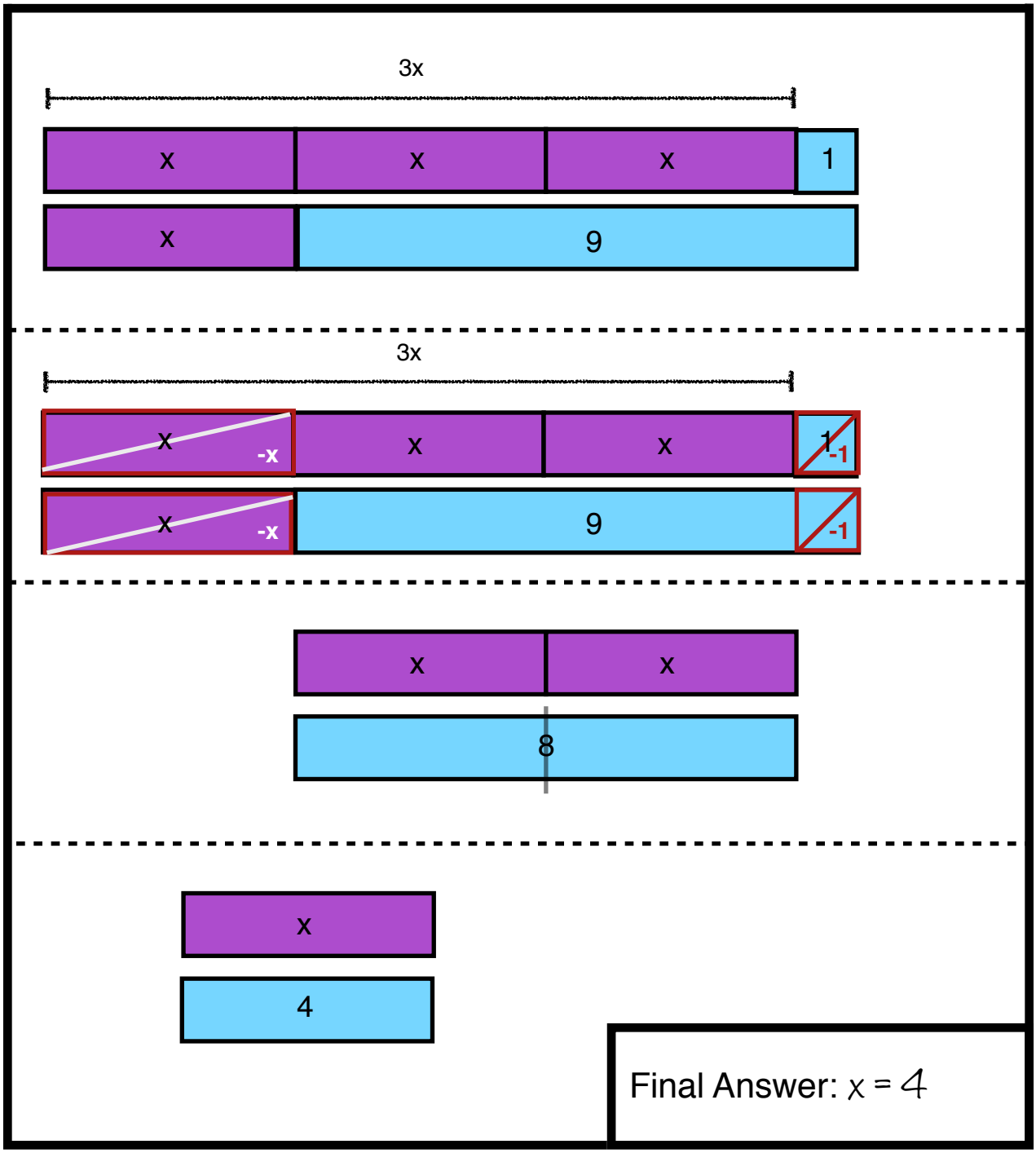
Problem: Frankie and Lana are both selling candy for a school fundraiser. Frankie sells three boxes of chocolate in addition to $12 worth of hard candies. Lana sells seven boxes of chocolate and brags that she has earned $4 more than Frankie. How much does each box of chocolates cost?



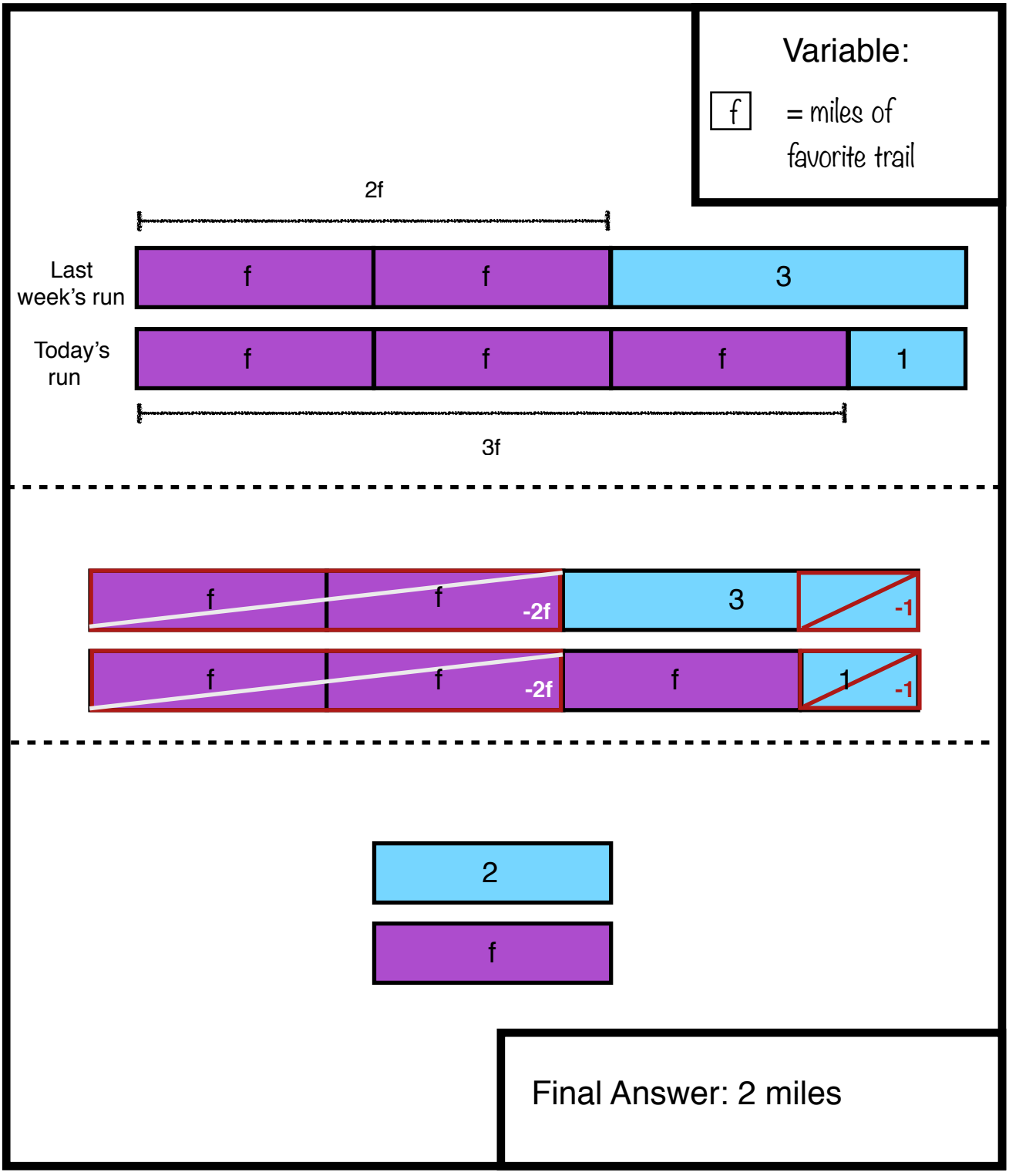
Problem: Monica and Bridget bought the same binders for school, and after filling them with school supplies, they weigh the same amount! Monica's binder contains a pencil pouch that weighs 2 pounds and 3 notebooks. Bridget's binder contains a stapler that weighs 1 pound, a pack of crayons that weighs 2 pounds, and 2 notebooks. How much does 1 notebook weigh?



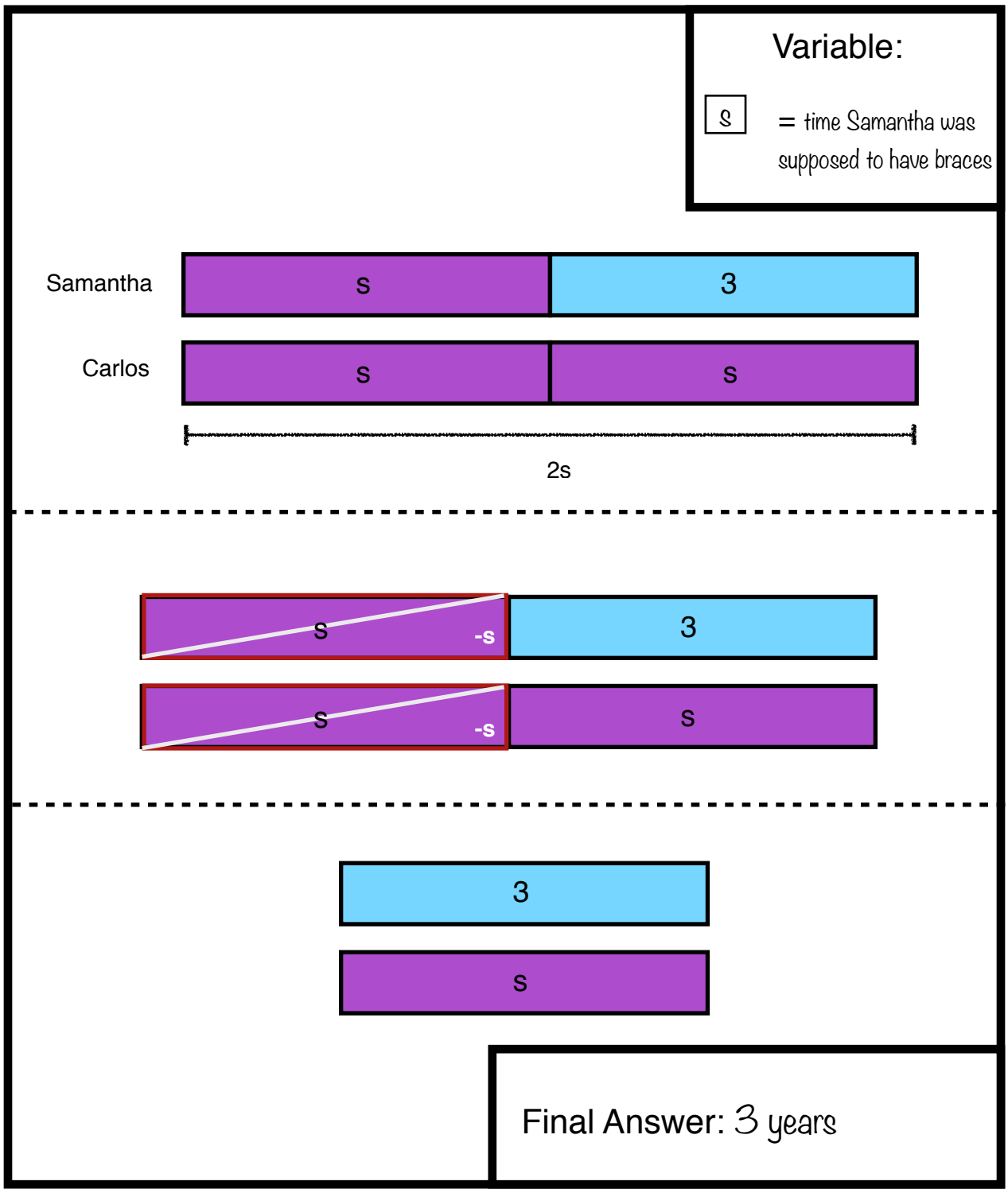
Problem: 3x+1=x+9



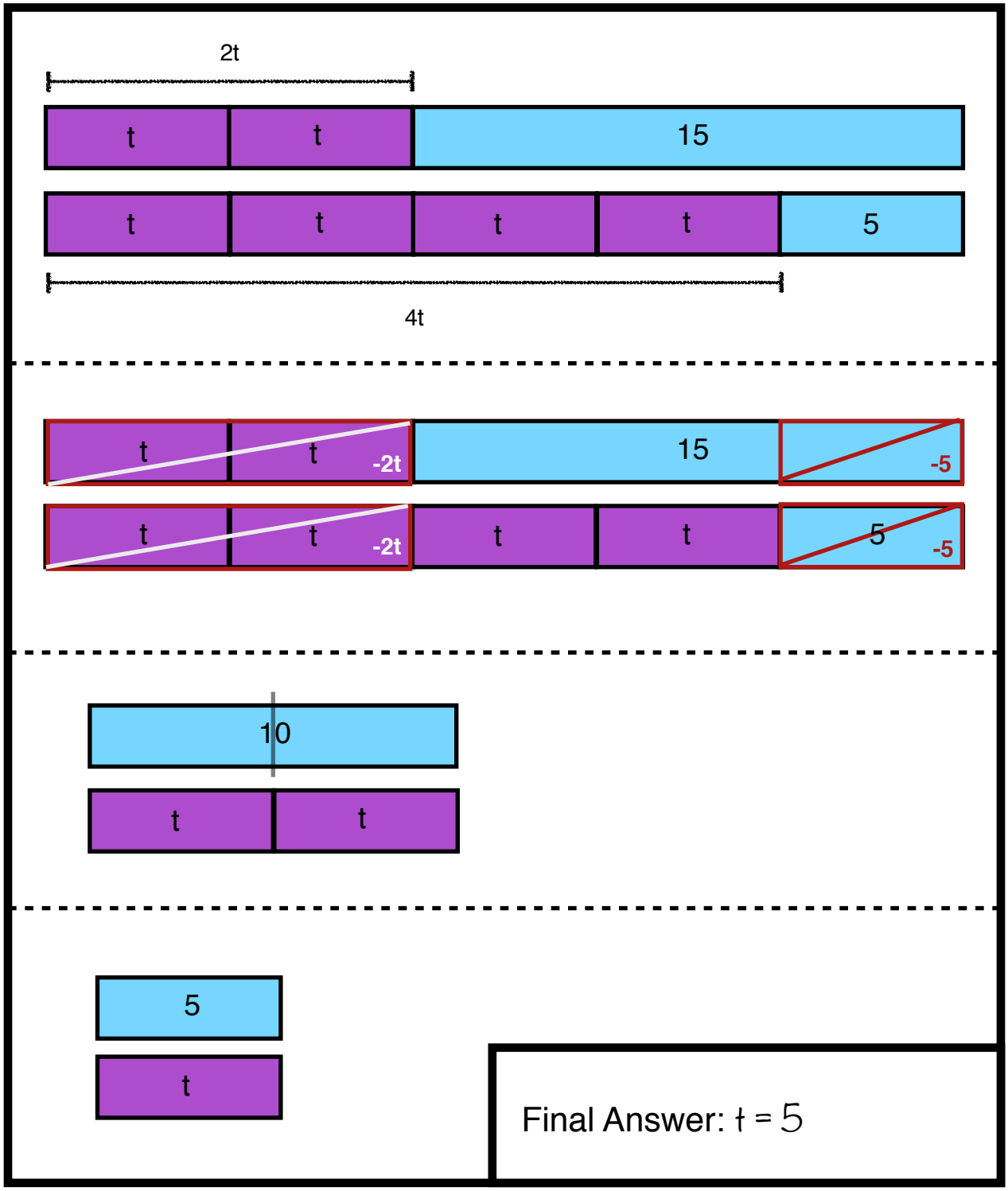
Problem: Jimmy always goes on runs that are the same distance. Last week he went on a run where he ran his favorite trail 2 times and then ran 3 miles to the park. Today, he ran his favorite trail 3 times and then ran 1 more mile. How many miles is his favorite trail?



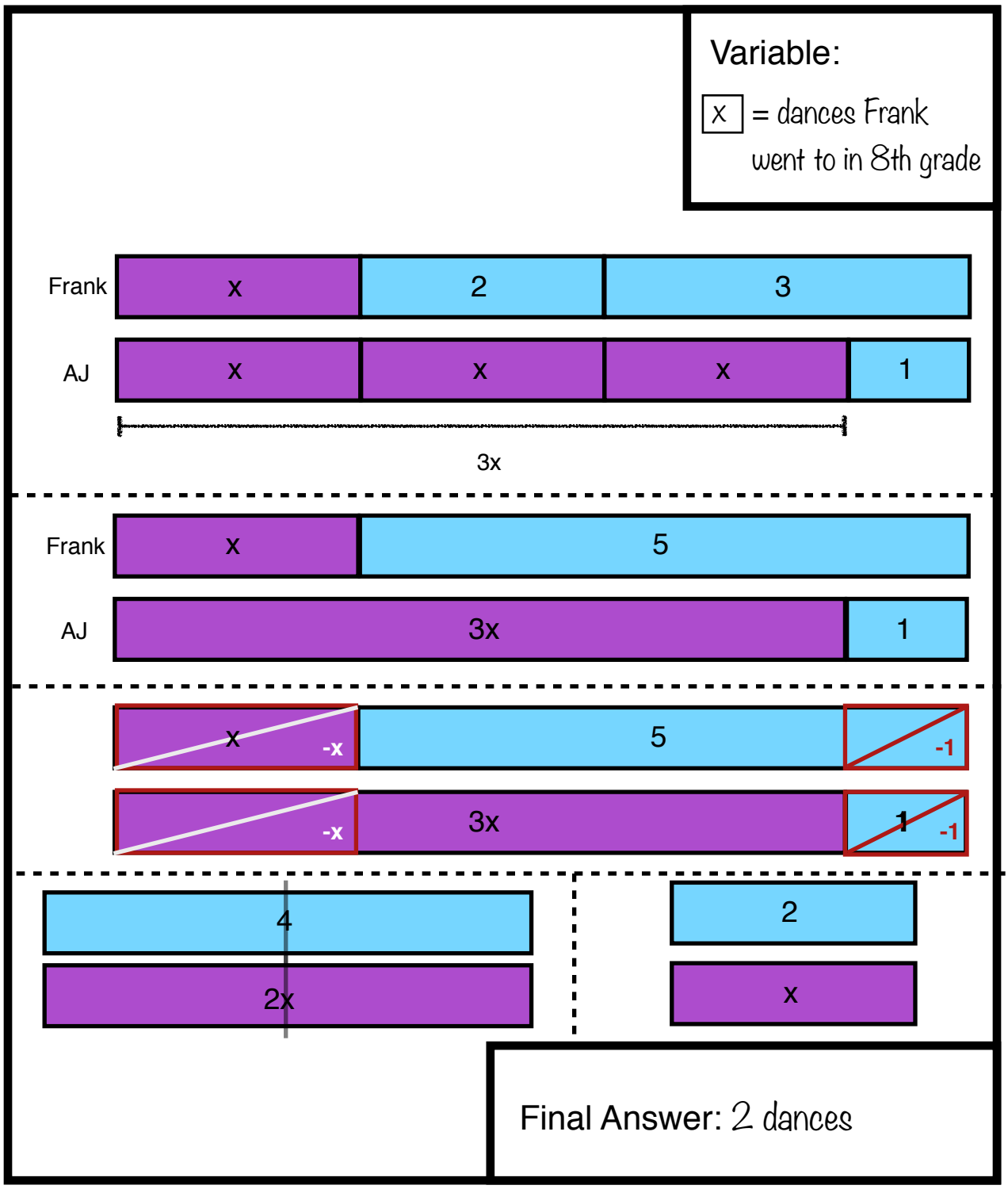
Problem: Samantha and Carlos wore braces for the same number of years. Samantha can't remember how many years her doctor said she would need braces for, but she knows she had braces for 3 years longer than the doctor expected. Carlos had braces for twice as long as Samantha was supposed to. How long was Samantha supposed to have braces for?



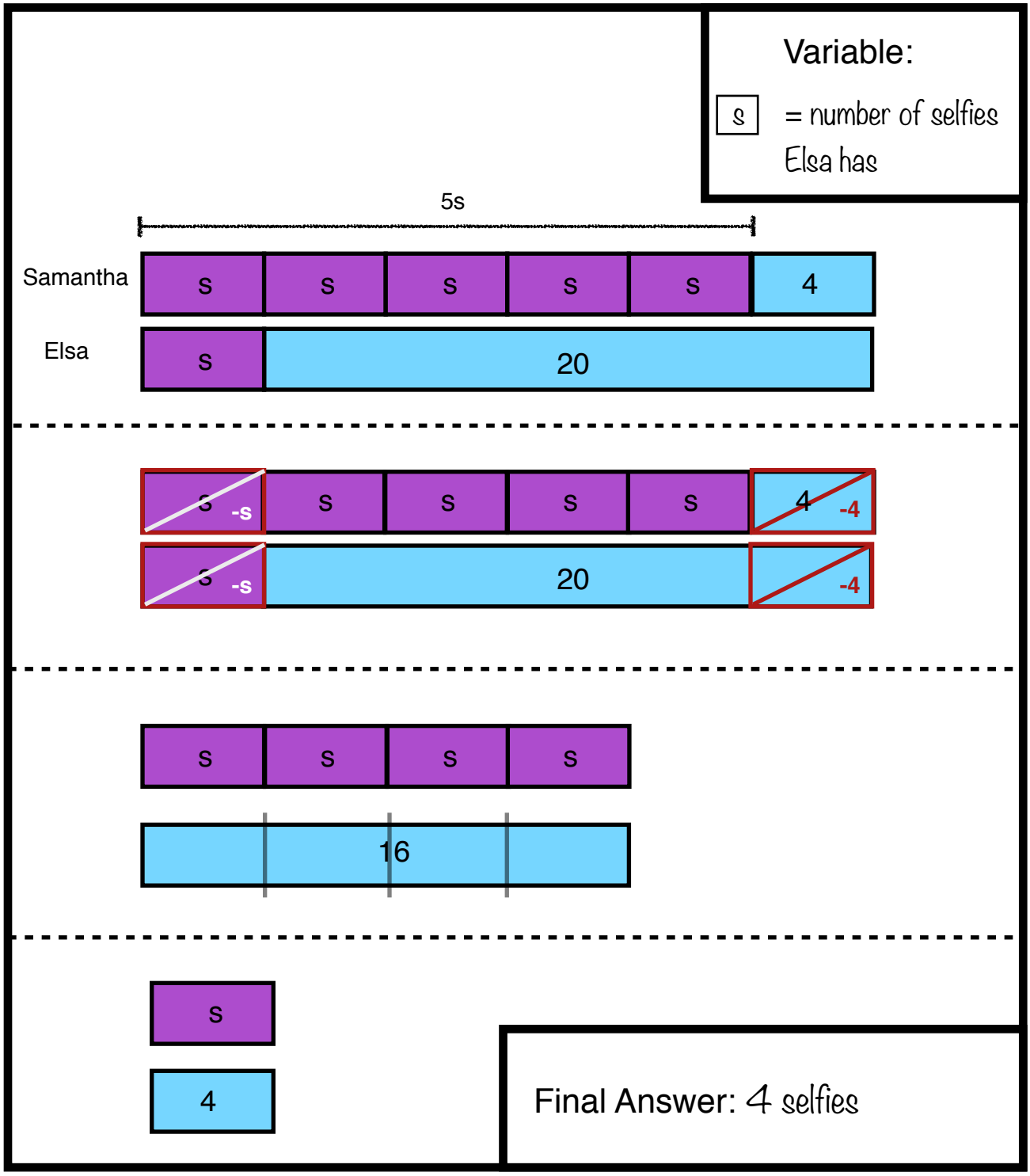
Problem: 2t +15 = 4t + 5



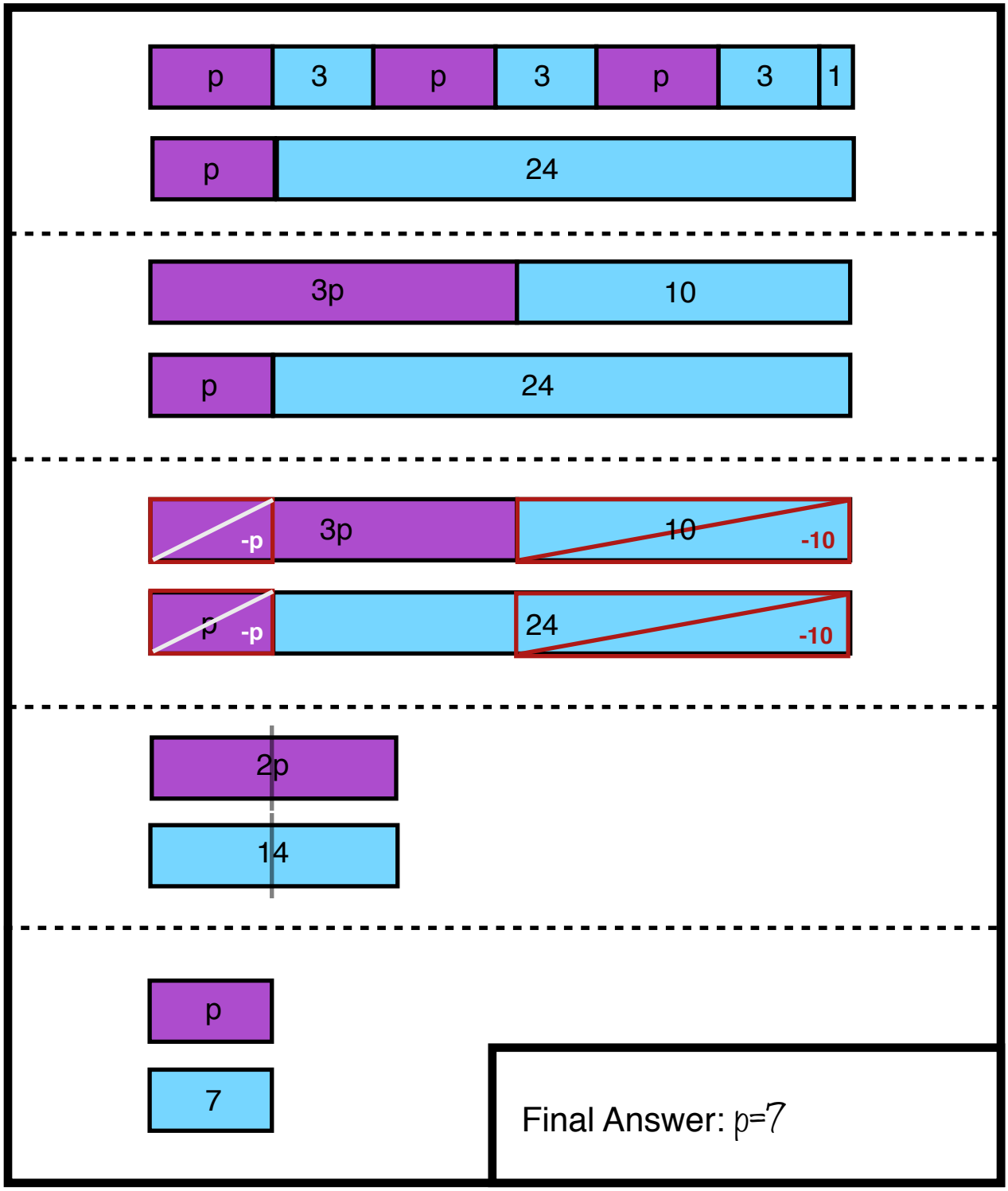
Problem: In middle school, Frank and AJ went to the same number of dances. Frank went to 2 dances in 6th grade, and 3 dances in 7th grade. AJ didn't go to any dances 6th grade, and went to to 1 dance in 7th grade. In 8th grade AJ went to three times as many dances as Frank. How many dances did Frank go to in 8th grade?



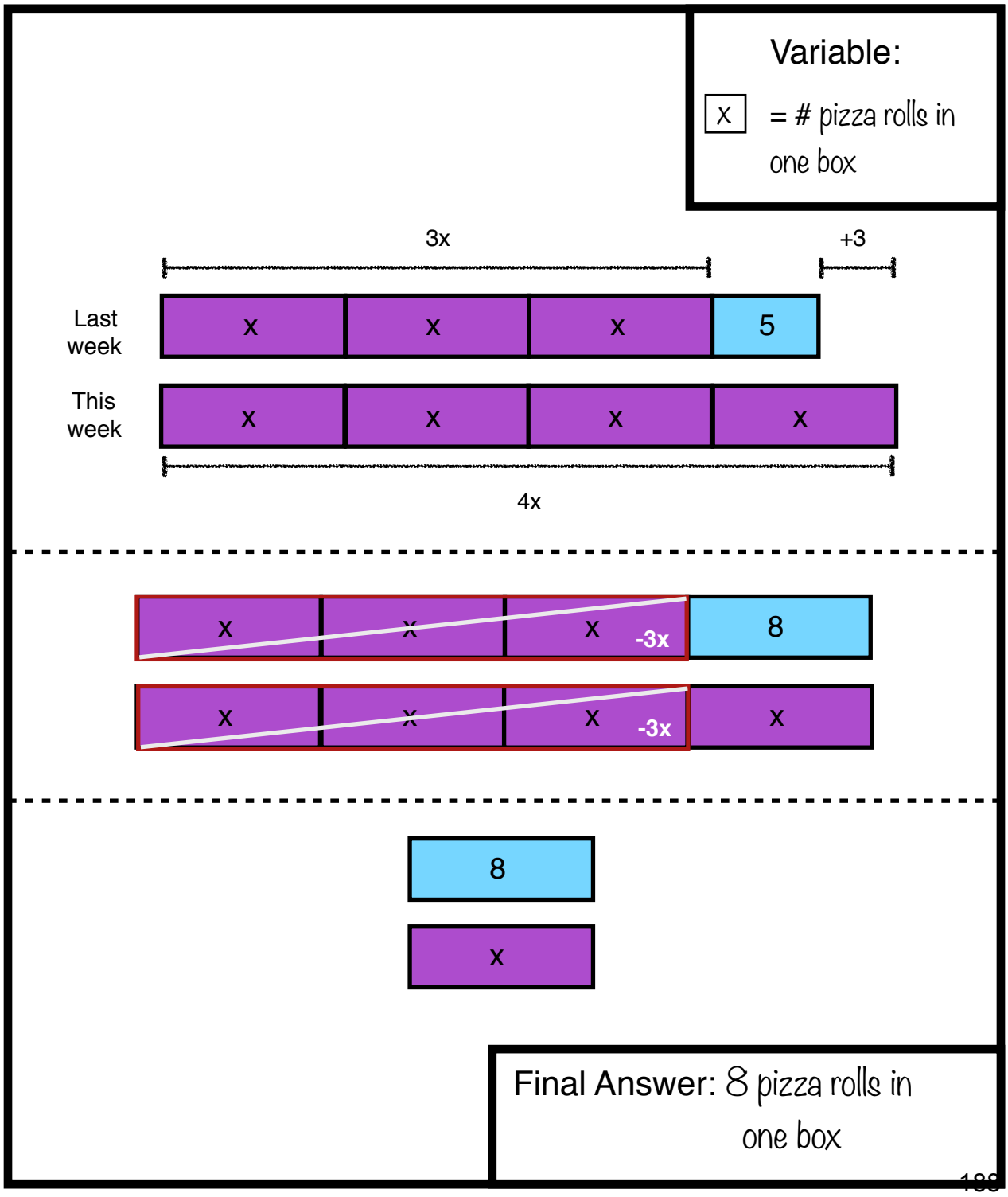
Problem: Samantha and Elsa have the same number of photos on their phones, and all of their pictures are either selfies or pictures of their pet. Samantha's phone has 5 times as many selfies as Elsa's. Elsa's phone has 20 pictures of her dog, and Samantha's phone has 4 pictures of her cat. How many selfies does Elsa have on her phone?



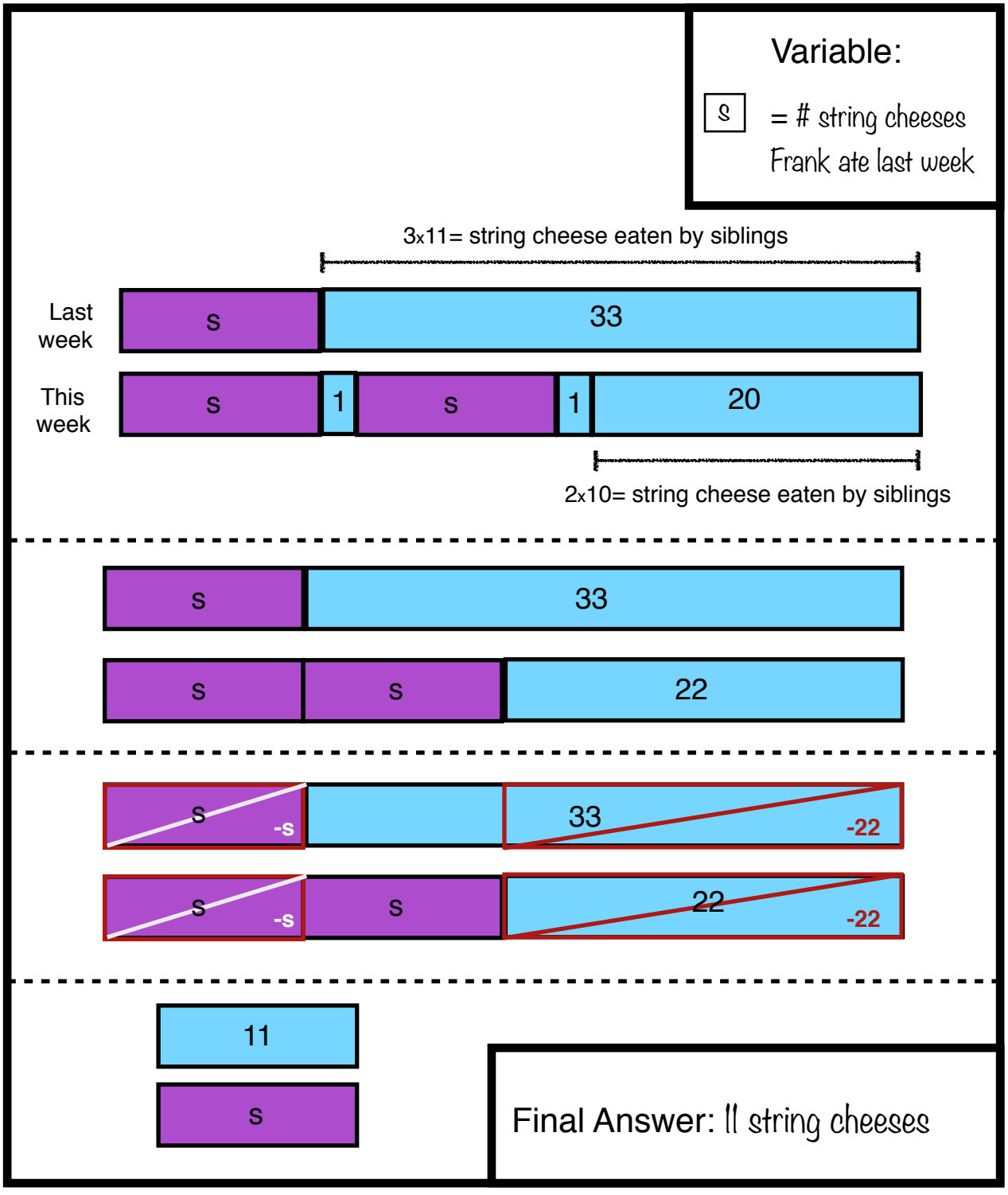
Problem: 3(p + 3) + 1 = p + 24



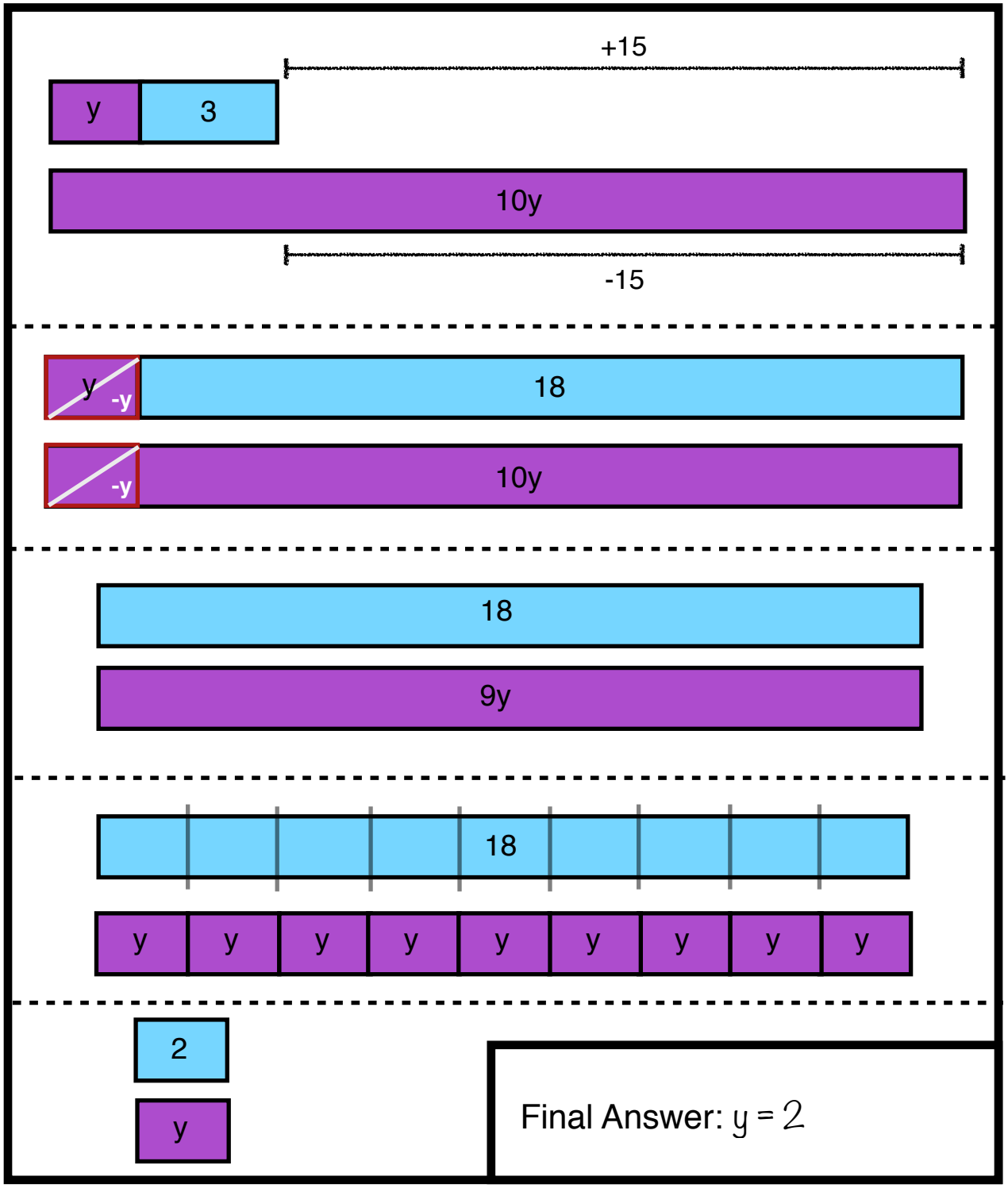
Problem: Every Friday, Lola has her friends over and they eat pizza rolls. Last week her friends made 3 boxes of pizza rolls and ate 5 pizza rolls that were leftover in the fridge. This week they ate 4 boxes of pizza rolls and ate 3 more rolls than they ate last week. How many pizza rolls are there in one box?



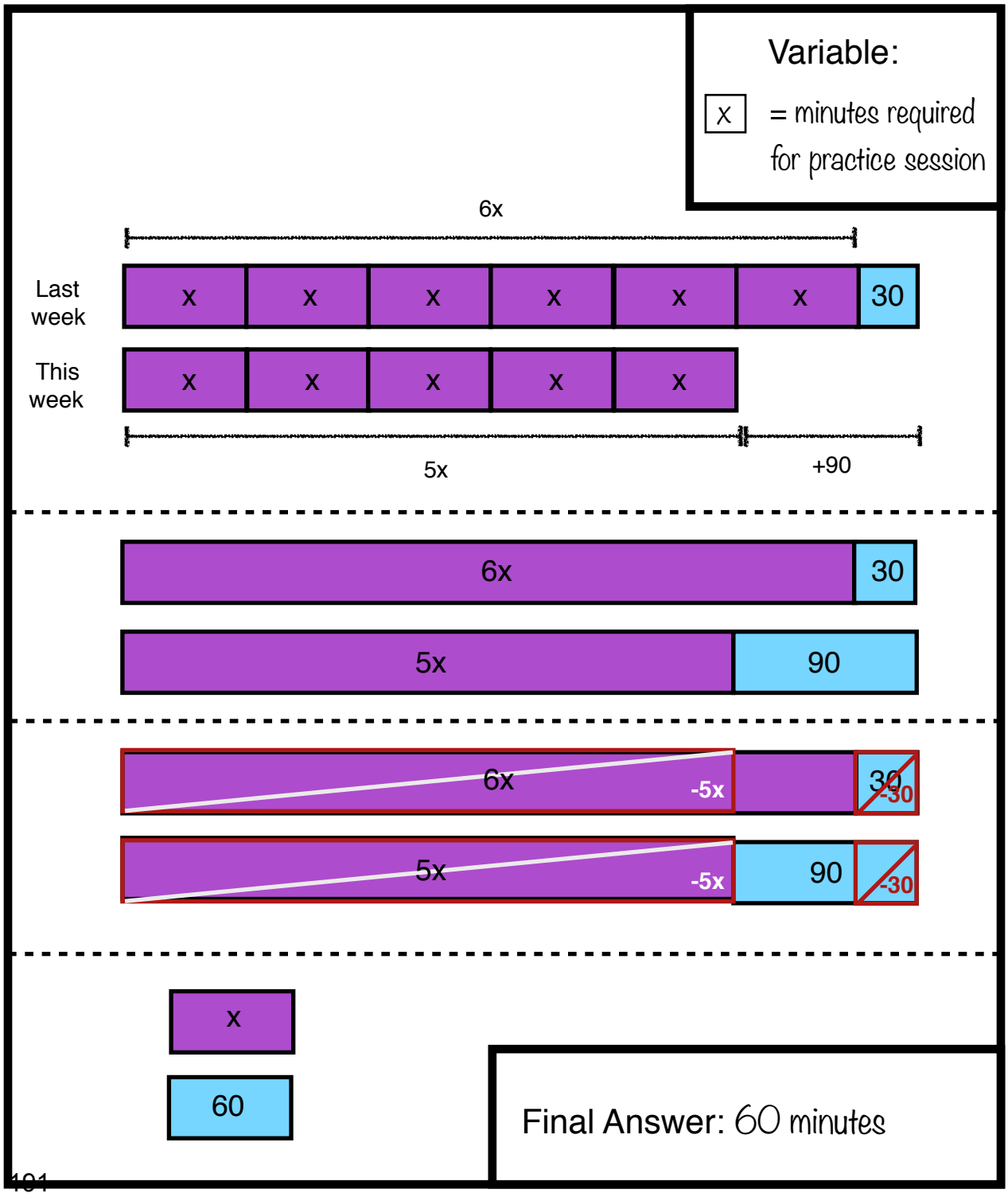
Problem: Frank has eleven siblings, and they all like to eat cheese sticks. Their dad buys the same number of cheese sticks every week. Last week, Frank ate many cheese sticks, and each of his siblings ate three cheese sticks. This week, Frank and his sister Sofia *each* ate one more than Frank had eaten the week before, and each of their *other* siblings ate two cheese sticks. How many cheese sticks did Frank eat last week?



Problem: y + 3 = 10y -15



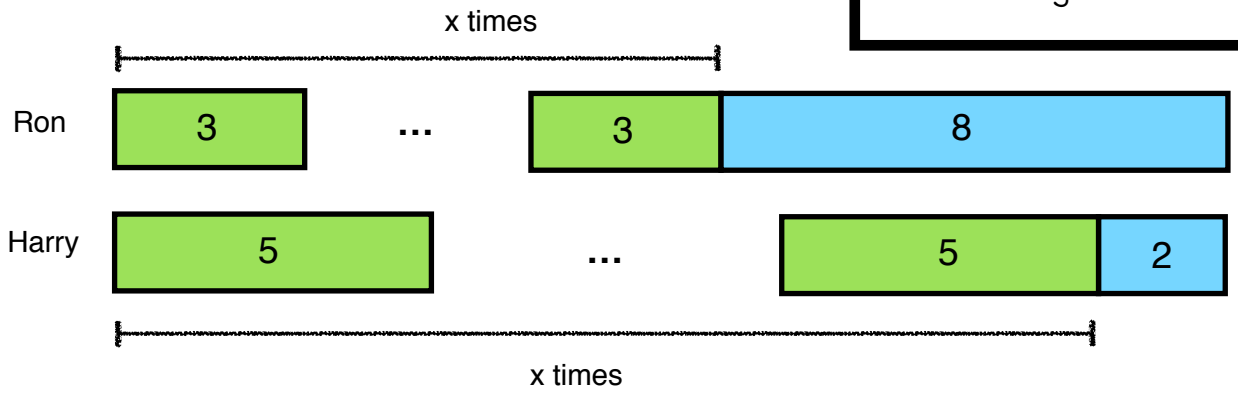
Problem: Gretchen plays the clarinet, and her teacher has a required amount of time that a practice session is supposed to last. Last week she practiced the required amount of time 6 times, and practiced for 30 extra minutes on Thursday. This week she practiced the required amount 5 times and practiced 90 minutes less this week than last week. How long does Gretchen's teacher require that a practice session last?



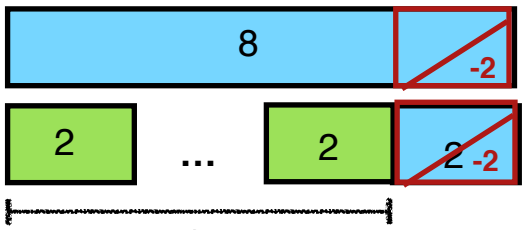
Problem: Ron and Harry love to tell jokes. On Tuesday Ron told 3 jokes in each class period and Harry told 5 jokes in each class period. Ron also told 8 jokes during lunch, and Harry told 2 jokes during lunch. If they only told jokes during class and at lunch, and they both told the same number of jokes on Tuesday, how many class periods were there on Tuesday?

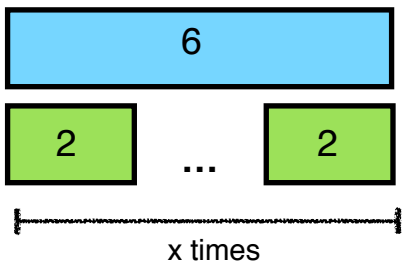
Variable:

x = # class periods Tuesday



In every class period, Ron tells 3 jokes for Harry's 5 jokes. So Harry tells 2 more jokes every class period. I can use that to simplify the model!



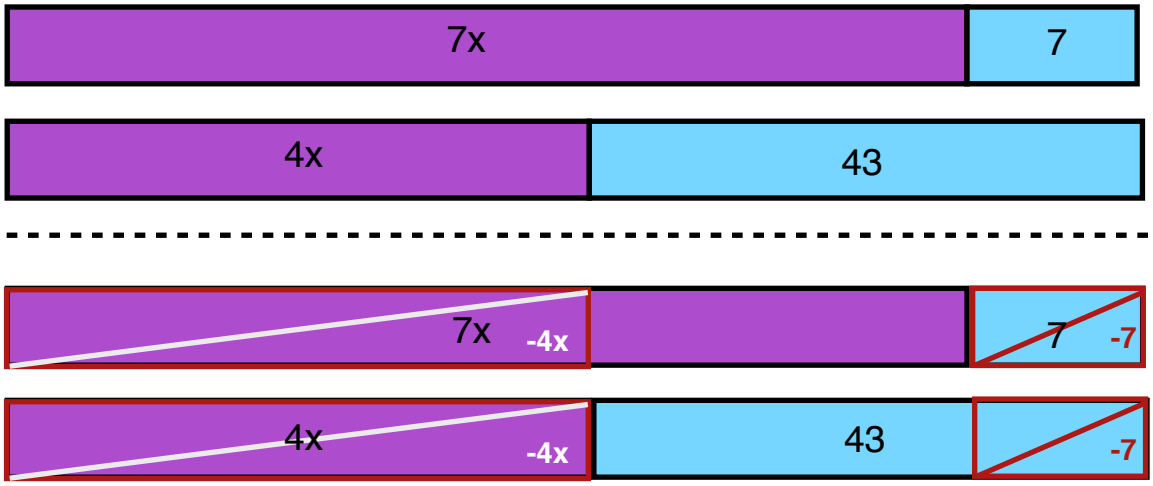


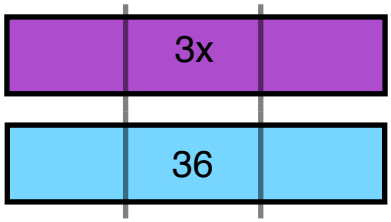
Final Answer: 3 periods

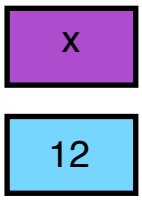
Problem: 7(x+1) = 4x + 43

drawing 7 “x” blocks and 7 “1” blocks is too much work, so I will use the distributive property to make it easier.

7(x+1) = 7x + 7







Final Answer: x = 12